CfE

Higher

Physics dept

Unit 1 Our Dynamic Universe

Summary Notes

1.1 Equations of Motion

Vectors

Vectors and Scalars (Revision of National 5)

It is possible to split up quantities in physics into two distinct groups, those that need a direction and those that don't. Some are obvious - it makes sense that force has direction; you can push or pull but you need to specify the direction.
It would be nonsense to give a direction to time. To say: "It took 5 seconds East" just isn't right. It is important that you are familiar with which quantity falls into which grouping.

A scalar is a quantity that can be described by just a **size** and a unit. e.g. time - 30 s, mass - 20 kg.

A vector is a quantity that is fully described with a **size and *direction***.
e.g. force - 50 N downwards velocity - 20 ms^{-1} East.

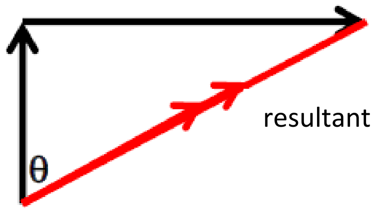
Adding Vectors (Revision of National 5)

This is more difficult than adding scalars as the direction of the vectors must be taken into account.
The addition of two vectors is called the ***resultant*** vector.
When you add vectors they have to be added ***tip-to-tail***.
What does this mean?

- Each vector must be represented by a straight line of suitable scale.
- The straight line must have an arrow head to show its direction. i.e.

- The vectors must be joined one at a time so that the tip of the previous vector touches the tail of the next vector. i.e.

- A straight line is drawn from the starting point to the finishing point and the starting angle is marked.

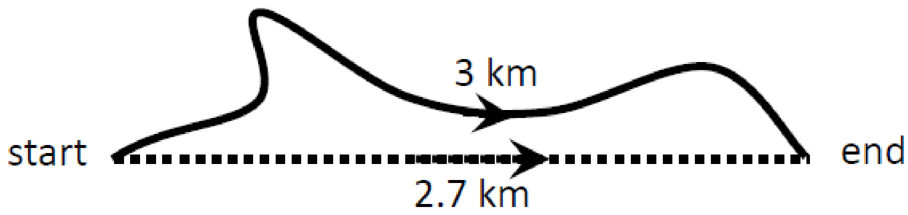


- The resultant should have 2 arrow heads to make it easy to recognise.
- If using a scale diagram the length and direction of this straight line gives the resultant vector.
- Alternatively you can use trigonometry and SOHCAHTOA or the sine or cosine rule to calculate the resultant.

Distance and Displacement (Revision of National 5)

The **distance** travelled by an object is the **sum of the distances** of each stage of the journey.
Since each stage has a different direction, the total distance has no single direction and therefore distance is a scalar.
The **displacement** of an object is the **shortest route between the start and finish point measured in a straight line**.
Displacement has a direction and is a vector.

Consider the journey below. A person walks along a path (solid line) from start to end.

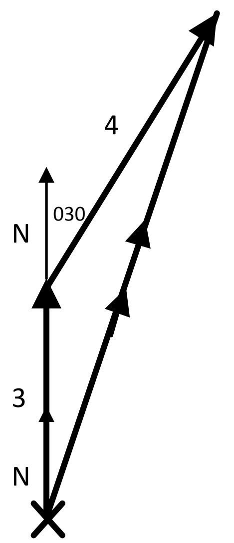


They will have walked further following the path than if they had been able to walk directly from start to end in a straight line (dashed line).
The solid line denotes the **distance** = 3km. The dashed line denotes the **displacement** = 2.7 km East

Example
A woman walks her dog 3 km due North (000) and then 4 km (030).
Find her
a) distance travelled
b) displacement.

Solution – **Use a ruler to measure the lengths of the vectors and a protractor to measure the bearing.**

- Choose an appropriate scale e.g. 1cm : 1km
- Mark the start point with an X, draw a North line and draw the first vector.
- Draw a North line at the tip of this vector and now draw the second vector (tip to tail)
- Draw the resultant vector from start to end using the double arrow.
- Measure the length of the line and the bearing.
  When measuring bearings remember - from START – CLOCKWISE – from NORTH



Speed and Velocity (Revision of National 5)

Speed is defined as the **distance travelled per second** and is measured in **metres per second**, **or ms^{-1}.**
Since distance and time are both scalar quantities then speed is also a scalar quantity.
From previous work in Maths and Physics we know that speed is calculated from the equation:

$$v = d / t$$

The **velocity** of an object is defined as **the displacement travelled per second**.
Since displacement is a vector quantity that means that velocity is also a vector and has the symbol **v**.
The equation for velocity is:

$$v = s / t$$

Example

A runner sprints 100 m East along a straight track in 12 s and then takes a further 13 s to jog 20 m back towards the starting point.

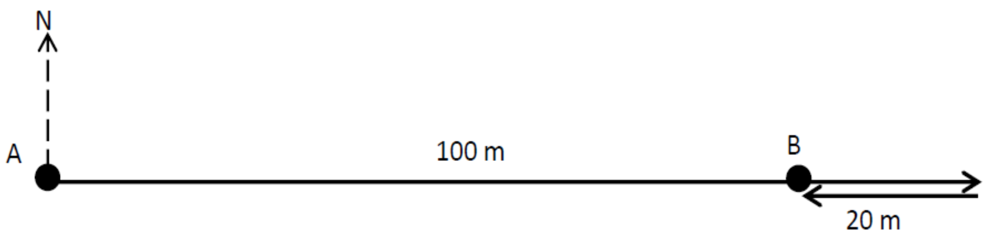
(a) What distance does she run during the 25 s?

(b) What is her displacement from her starting point after the 25 s?

(c) What is her speed?

(d)What is her velocity?

Solution – always draw the vector diagram.



a) d = 100 + 20
d = 120 m

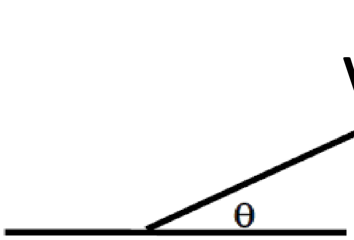
(b) s = 100 + (− 20)
s = 80 m (090)

c) v = d / t
v = 120/25
v = 4.8 ms^{-1}

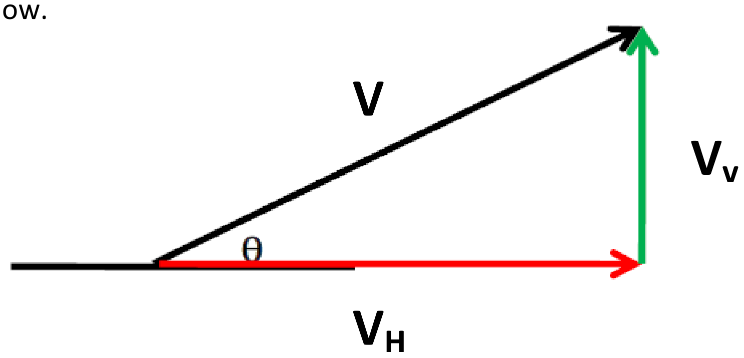
(d) v = s / t
v = 80/25
v = 3.2 ms^{-1} (090)

Resolving Vectors

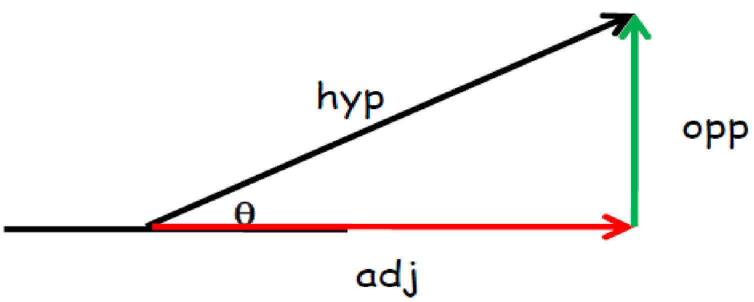
We have seen that two vectors can be added to give the resultant using vector addition.
Can we split a resultant vector into the two individual vectors that make it up?
Consider the following.



This shows a resultant vector, V, at an angle Θ to the horizontal.
To travel to the end of the vector we could move in a straight line in the X direction and then a straight line in the Y direction as shown below.



But how do we find out the size of each line?
Since we have a right angled triangle with a known angle we can name the sides.



This means we can use Pythagoras' theroem to work out the unknown sides.

horizontal component	vertical component
$\mathbf{V_H = V\cos\Theta}$	$\mathbf{V_V = V\sin\Theta}$

Example
A football is kicked at an angle of 70° at 15 ms^{-1}.
Calculate:

a) the horizontal component of the velocity;
b) the vertical component of the velocity.

Solution
$\mathbf{V_H = V\cos\Theta = 15\cos70 = 5.2\ ms^{-1}}$
$\mathbf{V_v = V\sin\Theta = 15\sin70 = 14.1\ ms^{-1}}$

The 3 Equations of Motion

The 3 Equations of Motion (s u v a t)

The equations of motion can be applied to any object moving with constant acceleration in a straight line. You must be able to:

- select the correct formula;
- identify the symbols and units used;
- carry out calculations to solve problems of real life motion; and
- carry out experiments to verify the equations of motion.

You should develop an understanding of how the graphs of motion can be used to derive the equations. This is an important part of demonstrating that you understand the principles of describing motion, and the link between describing it graphically and mathematically.

Equation of Motion 1 : *v = u + at*

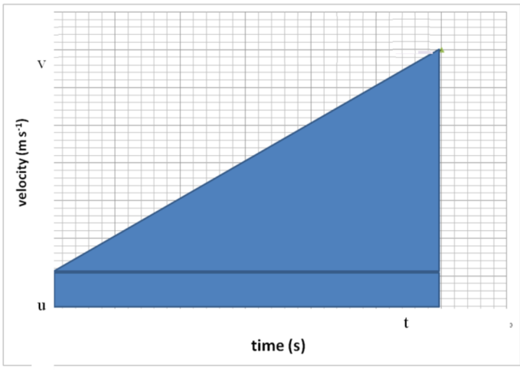
$$a = \frac{v-u}{t}$$

at = v – u

u + at = v

OR

v = u + at



Example

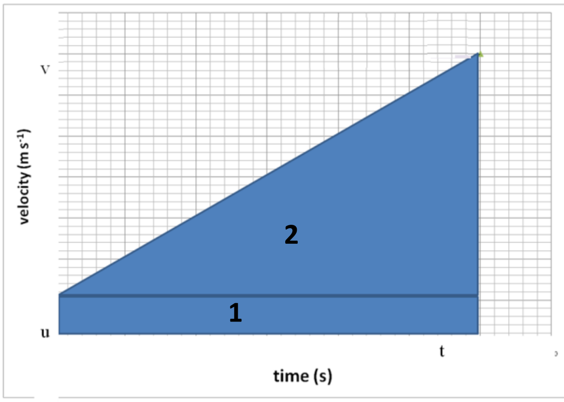
A racing car starts from rest and accelerates uniformly in a straight line at 12 ms^{-2} for 5.0 s. Calculate the final velocity of the car.

Solution **LIST s u v a t**

S	
u = 0 ms^{-1} (rest)	
v	v = u + at
a = 12 ms^{-2}	v = 0 + (12 x5.0)
t = 5.0 s	v = 0 + 60
	v = 60 ms^{-1}

The 3 Equations of Motion (ctd)

Equation of Motion 2: $s = ut + \frac{1}{2} at^2$



The displacement, s, is the area under the graph:

Area 1 = ut

Area 2 = ½ (v-u) t

But from equation 1 we get that (v-u) = at

So Area 2 = ½ (at)t

Therefore,

$$s = ut + \frac{1}{2}(at)t$$

OR

$$\mathbf{s = ut + \frac{1}{2} at^2}$$

<u>Example</u>

A speedboat travels 400 m in a straight line when it accelerates uniformly from 2.5 ms^{-1} in 10 s. Calculate the acceleration of the speedboat.

<u>Solution</u>

s = 400 m	$s = ut + \frac{1}{2} at^2$
$u = 2.5\ ms^{-1}$	$400 = (2.5 \times 10) + (0.5 \times a \times 10^2)$
v	$400 = 25 + 50a$
a = ?	$50a = 400 - 25 = 375$
t = 10 s	$a = 375/50$
	$a = 7.5\ ms^{-2}$

Equation of Motion 3: $v^2 = u^2 + 2as$

We have already found that

$$v = u + at$$

(square both sides) $v^2 = (u + at)^2$

$$v^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2a(ut + \frac{1}{2} at^2)$$

And since $s = ut + \frac{1}{2} at^2$

$$\mathbf{v^2 = u^2 + 2as}$$

Example

A rocket is travelling through outer space with uniform velocity. It then accelerates at 2.5 ms^{-2} in a straight line in the original direction, reaching 100 ms^{-1} after travelling 1875 m.

Calculate the rocket's initial velocity.

Solution

s = 1 875 m	$v^2 = u^2 + 2as$
u = ?	$100^2 = u^2 + (2 \times 2.5 \times 1875)$
$v = 100\ ms^{-1}$	10 000 = u2 +9375
$a = 2.5\ ms^{-2}$	$u^2 = 10\ 000 - 9375$
t	$u = 25 ms^{-1}$

The 3 Equations of Motion with Decelerating Objects

When an object decelerates its velocity decreases. If the vector quantities in the equations of motion are positive, we represent the decreasing velocity by use of a negative sign in front of the acceleration value.

Example 1

A car, travelling in a straight line, decelerates uniformly at 2.0 ms^{-2} from 25 ms^{-1} for 3.0 s.
Calculate the car's velocity after the 3.0 s.

Solution

s	
u = 25 ms^{-1}	v = u + at
v = ?	v = 25 + (**-2.0** x 3.0)
a = -2.0 ms^{-2}	v = 25 + (-6.0)
t = 3.0 s	v = 19 ms^{-1}

Example 2

A greyhound is running at 6.0 ms^{-1}. It decelerates uniformly in a straight line at 0.5 ms^{-2} for 4.0 s.
Calculate the displacement of the greyhound while it was decelerating.

Solution

s = ?	s = ut + ½ at^2
u = 6.0 ms^{-1}	s = (6.0 x 4.0) + (0.5 x **-0.5** x 4.0^2)
v	s = 24 + (-4.0)
a = -0.5 ms^{-2}	s = 20 m
t = 4.0 s	

Example 3

A curling stone leaves a player's hand at 5.0 ms^{-1} and decelerates uniformly at 0.75ms^{-2} in a straight line for 16.5 m until it strikes another stationary stone.
Calculate the velocity of the decelerating curling stone at the instant it strikes the stationary one.

Solution

s = 16.5 m	$v^2 = u^2 + 2as$
u = 5.0 ms^{-1}	$v^2 = 5.0^2 + (2 \times -0.75 \times 16.5)$
v = ?	v^2 = 25 + (-24.75)
a = -0.75 ms^{-2}	v = √0.25
t	v = 0.5 ms^{-1}

Graphing Motion

Graphs

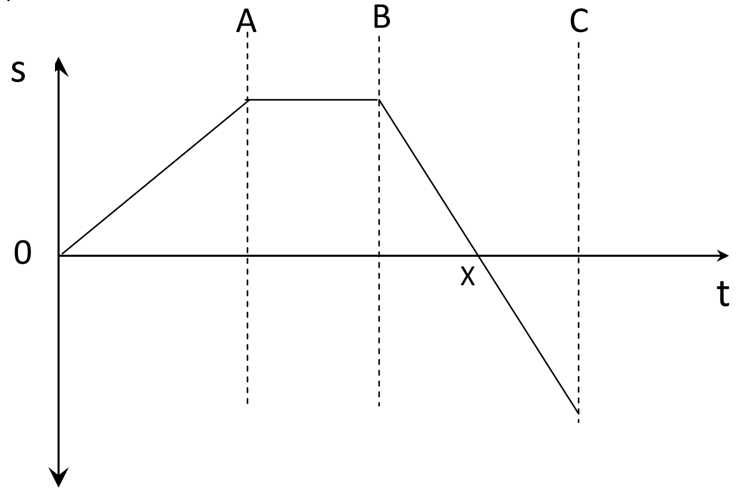
In all areas of science, graphs are used to display information.
Graphs are an excellent way of giving information, especially to show relationships between quantities.
In this section we will be examining three types of motion-time graphs.

Displacement-time graphs
Velocity-time graphs
Acceleration-time graphs

If you have an example of one of these types of graph then it is possible to draw a corresponding graph for the other two factors.

Displacement – time graphs

This graph represents how far an object is from its starting point at some known time. Because displacement is a vector it can have positive and negative values. (+ve and –ve will be opposite directions from the starting point).



OA – the object is moving away from the starting point. It is moving a constant displacement each second. This is shown by the constant gradient. What does this mean?

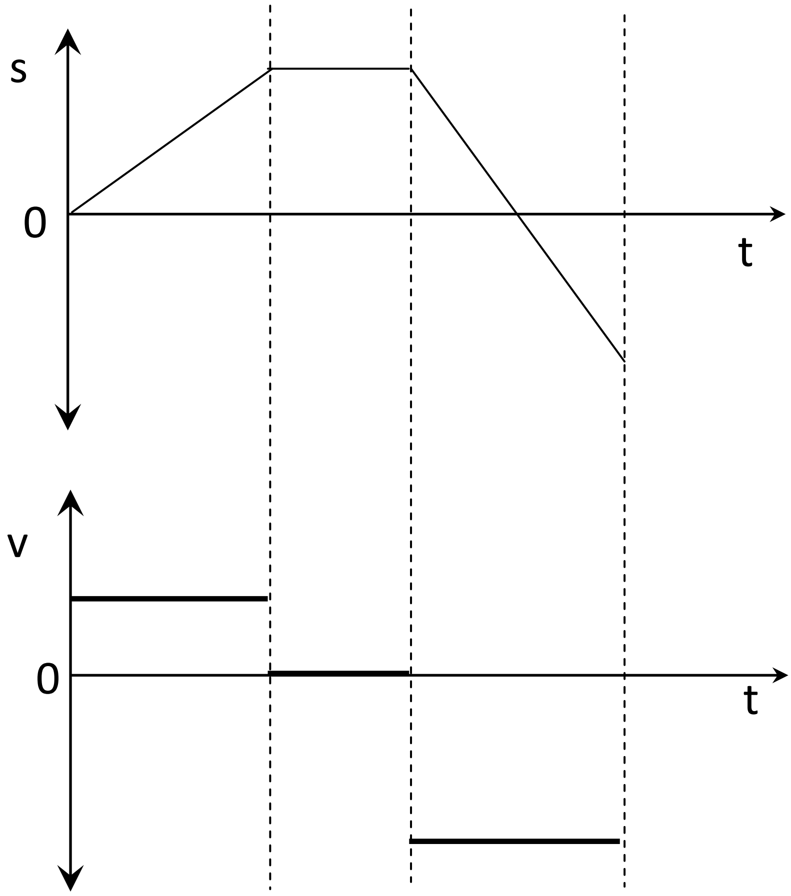
$$\textbf{gradient} = \frac{\textbf{displacement}}{\textbf{time}} = \textbf{velocity}$$

We can determine the velocity from the gradient of a displacement time graph.

AB – the object has a constant displacement so is not changing its position, therefore it must be at rest. The gradient in this case is zero, which means the object has a velocity of zero [at rest]

BC – the object is now moving back towards the starting point, reaching it at time x. It then continues to move away from the start, but in the opposite direction. The gradient of the line is negative, indicating the change in direction of motion.

Converting Displacement – time Graphs to Velocity-time Graphs



The velocity time graph is essentially a graph of the gradient of the displacement time graph. It is important to take care to determine whether the gradient is positive or negative.

The gradient gives us the information to determine the direction an object is moving.

There are no numerical values given on the graphs above. Numbers are not needed to allow a description. The will need to be used however if we were to attempt a quantitive analysis.

Velocity – time Graphs

It is possible to produce a velocity time graph to describe the motion of an object. All velocity time graphs that you encounter in this course will be of objects that have constant acceleration.

Scenario: The Bouncing Ball

Lydia fires a ball vertically into the air from the ground. The ball reaches its maximum height, falls, bounces and then rises to a new, lower, maximum height.

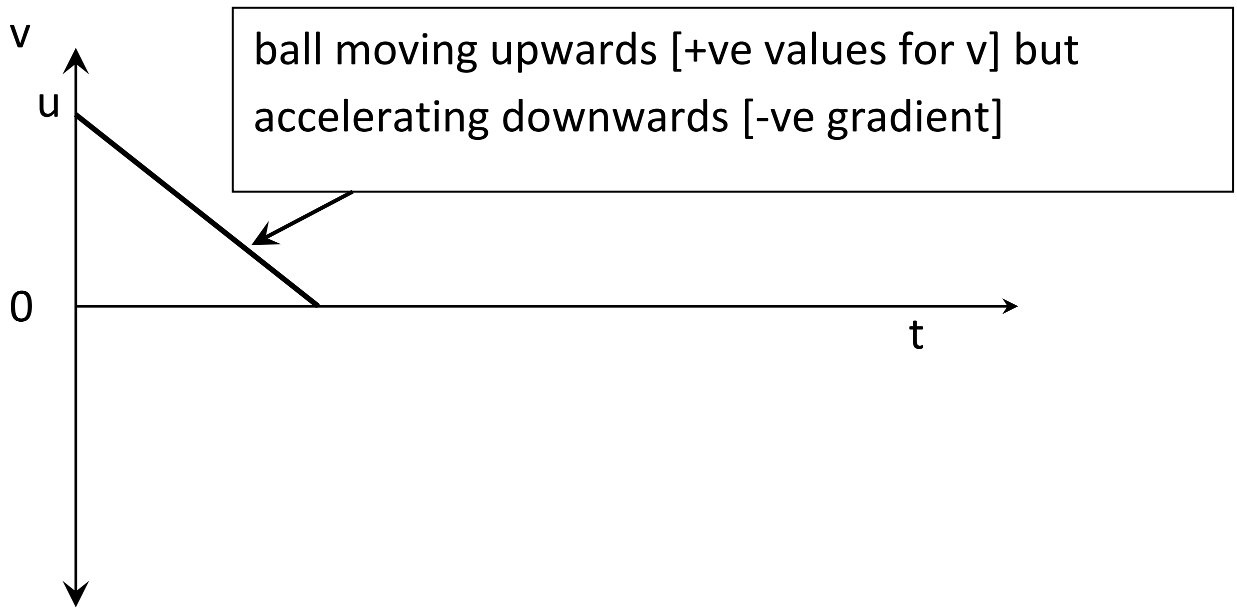
What will the velocity time graph for this motion look like?

First decision: The original direction of motion is up so upwards is the positive direction

Part One of Graph

Now we need to think, what is happening to the velocity?

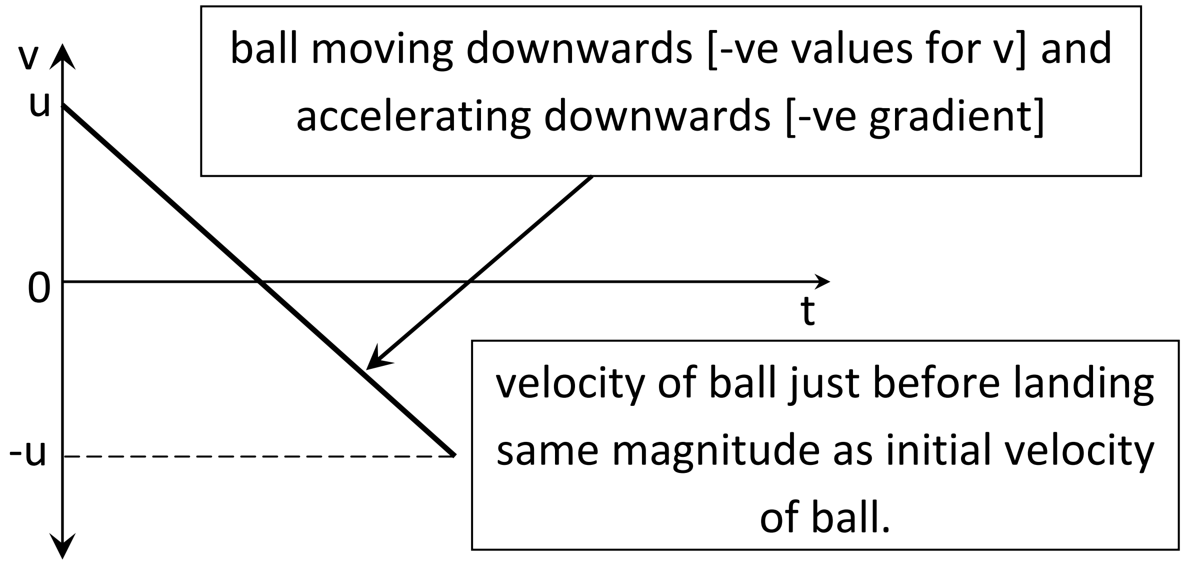
The ball will be slowing down whilst it is moving upwards, having a velocity of zero when it reaches maximum height. The acceleration of the ball will be constant if we ignore air resistance.



Velocity – time Graphs (continued)

Part Two of Graph

Once the ball reaches its maximum height it will begin to fall downwards. It will accelerate at the same rate as when it was going up. The velocity of the ball just before it hits the ground will be the same magnitude as its initial velocity upwards

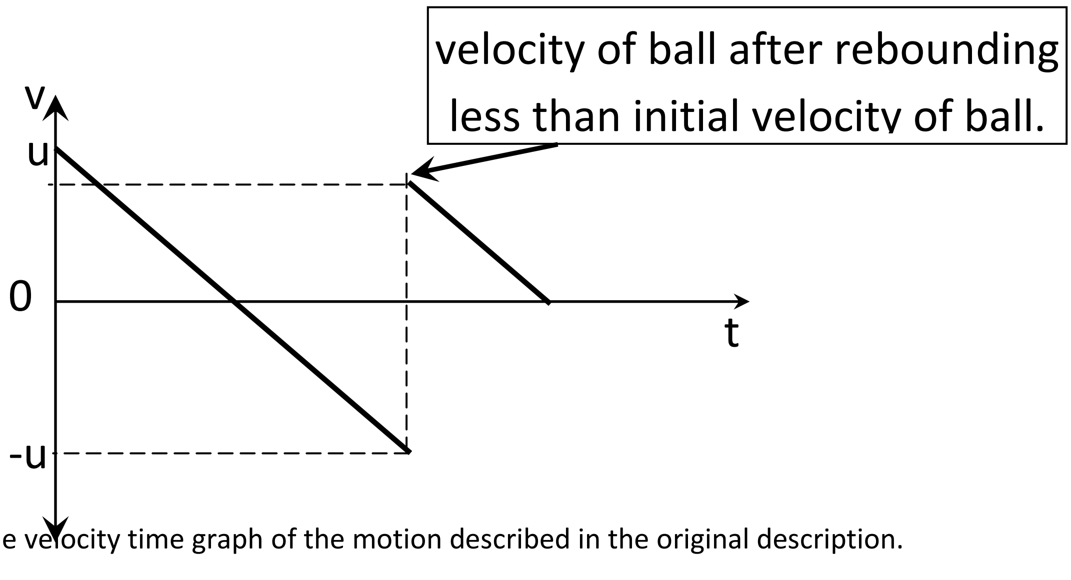


Part Three of Graph

The ball has now hit the ground. At this point it will rebound and begin its movement upwards.

In reality there will be a finite time of contact with the ground when the ball compresses and regains its shape. In this interpretation we will regard this time of contact as zero. This will result in a disjointed graph.

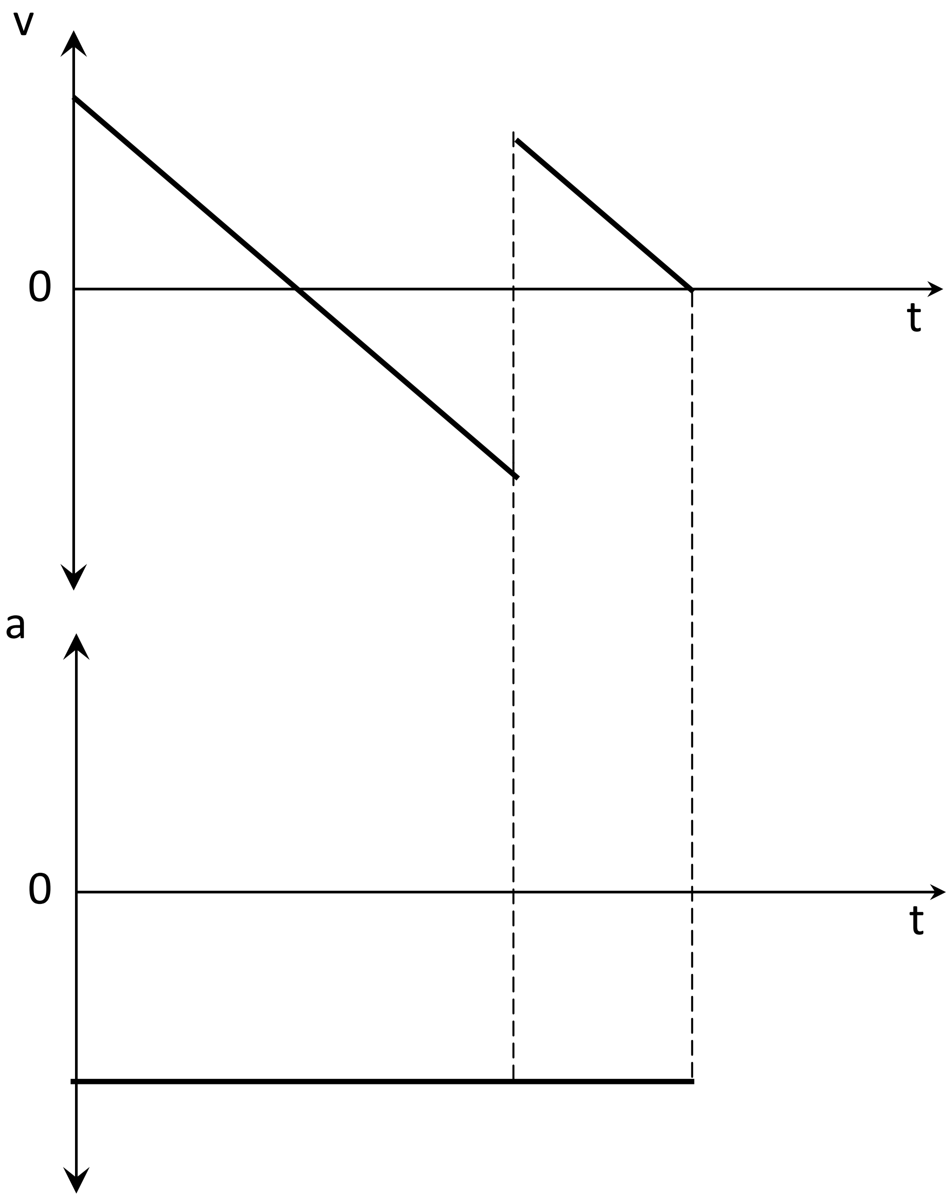
The acceleration of the ball after rebounding will be the same as the initial acceleration. The two lines will be parallel.



This now is the velocity time graph of the motion described in the original description.

Converting Velocity – time Graphs to Acceleration – time Graphs

What is important in this conversion is to consider the gradient of the velocity-time graph line.
In our example the gradient of the line is constant and has a **negative** value. This means for the entire time sampled the acceleration will have a single **negative** value.



All acceleration time graphs you are asked to draw will_consist of horizontal lines, either above, below or on the time axis.

Reminder from National 5

The area under a speed time graph is equal to the distance travelled by the object that makes the speed time graph.

In this course we are dealing with vectors so the statement above has to be changed to:

The area under a **velocity** time graph is equal to the **displacement of** the object that makes the speed time graph.

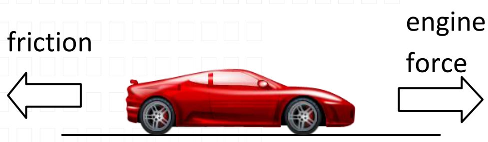
Any calculated areas that are below the time axis represent **negative** displacements.

1.2 Forces, Energy and Power

Forces

Newton's 1st Law of Motion (Revision of National 5)

An object will remain at rest or travel in a straight line at a constant velocity (or speed) if the forces are balanced.



- If we consider the car moving in a straight line. If the engine force = friction, it will continue to move at a constant velocity (or speed) in the same direction.
- If the same car is stationary (not moving) and all forces acting on it are balanced (same as no force at all) the car will not move.

Newton's 2nd Law of Motion (Revision of National 5)

This law deals with situations when there is an unbalanced force acting on the object.

The velocity cannot remain constant and the acceleration produced will depend on:

- the mass (**m**) of the object **($a \propto 1/m$) - if m increases a decreases and vice versa;**
- the unbalanced force **(F)** **($a \propto F$) - if F increases a increases and vice versa.**

This law can be summarised by the equation $F = ma$

Newton's 3rd Law of Motion (Revision of National 5)

Newton noticed that forces occur in pairs. He called one force the **action** and the other the **reaction**. These two forces are always equal in size, but opposite in direction. They do not both act on the same object (do not confuse this with balanced forces).

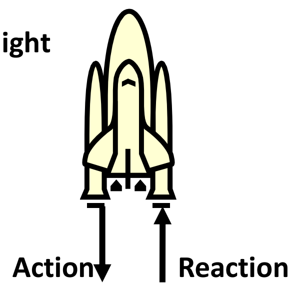
Newton's Third Law can be stated as:
If an object A exerts a force (the action) on object B, then object B will exert an equal, but opposite force (the reaction) on object A.

For example:

a) Kicking a ball

Action: The foot exerts a force on the ball to the right
Reaction: The ball exerts an equal force on the left to the foot

b) Rocket flight



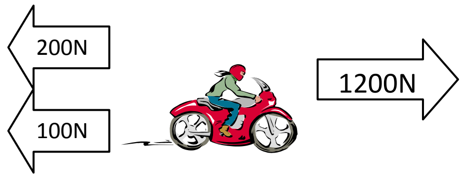
Action: The rocket pushes gases out the back
Reaction: The gases push the rocket in the opposite direction.

Resultant Forces – Horizontal

When several forces act on one object, they can be replaced by one force which has the same effect. This single force is called the **resultant** or **unbalanced** force. Remember that **Friction** is a resistive force which acts in the **opposite** direction to motion.

Example: Horizontal
A motorcycle and rider of combined mass 650 kg provide an engine force of 1200 N. The friction between the road and motorcycle is 100N and the drag value = 200N.
Calculate:

a) the unbalanced force acting on the motorcycle
b) the acceleration of the motorcycle



Solution

a) Draw a free body diagram

F = 1200 - (200 + 100)
F = 900 N   This 900 N force is the resultant of the 3 forces

b) F = 900 N   F = ma
a = ?   900 = 650 x a
m = 650 kg   a = 1.38 ms^{-2}

Resultant Forces – Vertical (Rocket)

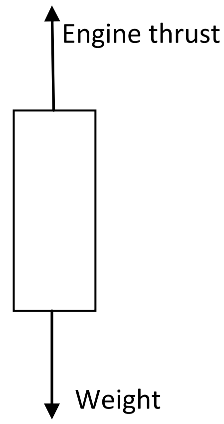
Example

At launch, a rocket of mass 20 000 kg accelerates off the ground at 12 ms^{-2} (ignore air resistance)

a) Use Newton's 3rd law of motion to explain how the rocket gets off the ground.
b) Draw a free body diagram to show all the vertical forces acting on the rocket as it accelerates upwards.
c) Calculate the engine thrust of the rocket which causes the acceleration of 12ms^{-2}.

Solutions

a) The rocket pushes the gas out the back downwards (action) and the gas pushes the rocket upwards (reaction).
b)

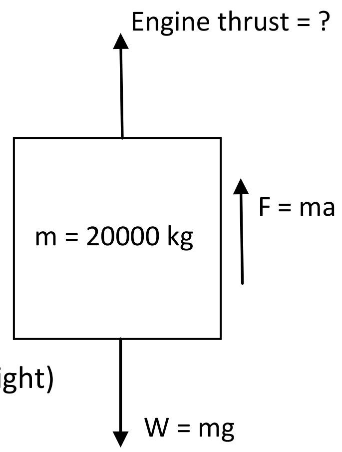


c) Calculate F and W

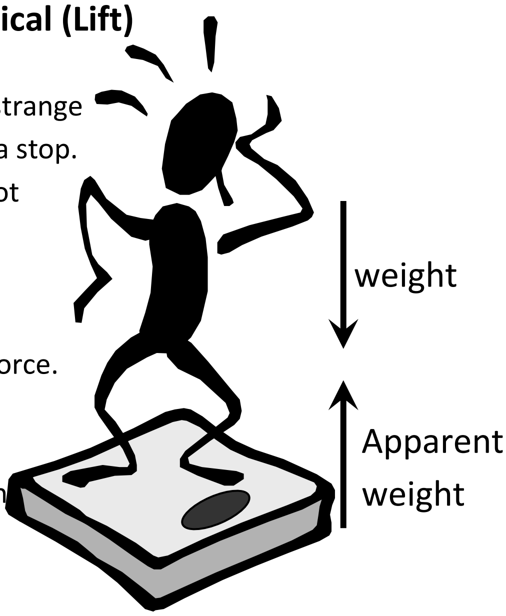
F = m a
F = 20 000 x 12
F = 240 000 N

W = m g
W = 20 000 x 9.8
W = 196 000 N

F = upward force (thrust) – downwards force (Weight)
240 000 = thrust – 196 000
<u>thrust = 436 000 N</u>



Resultant Forces – Vertical (Lift)



Have you noticed that when you are in a lift you experience a strange feeling when the lift starts to move and as it begins to slow to a stop. However when the lift is in the middle of its journey you cannot tell if you are moving at all.

This is because at the start and end of the journey you will experience an acceleration and consequently an unbalanced force. This unbalanced force is what you 'feel'.

When you stand on a set of scales (Newton Balance) the reading on the scales is actually measuring the **upwards** force.
This is the force the scales exert on you.
We will call this the **Apparent Weight**.
Now this is fine when you are in your bathroom trying to find your weight.

Normally, you and your bathroom scales will be stationary and so your weight will be equal to the upwards force (balanced forces).

When you weigh yourself when you are accelerating the reading on the scales will **not** be your weight. The reading will give you an indication of the unbalanced force acting on you, which could then be used to calculate an acceleration. This unbalanced force could be acting up or down depending on the magnitude and direction of the acceleration.

The value of the Apparent Weight will be equal to the Tension T in the cable of the lift.

The table below explains the motion of a lift as it moves from an upper floor to a lower floor and then back to the original floor.

Motion	Comparing W and T	Unbalanced Force F	Sensation in lift
Stationary at top	W=T	F = 0	
Accelerating down	T decreases W > T	F = W – T	Lighter
Constant velocity down	T increases W = T	F = 0	
Decelerating down	T increases T > W	F = T – W	Heavier
Stationary at bottom	T decreases T = W	F = 0	
Accelerating up	T increases T > W	F = T – W	Heavier
Constant speed up	T decreases T = W	F = 0	
Decelerating up	T decrease W > T	F = W - T	Lighter

Resultant Forces – Vertical (Lift continued)

Example

A man of mass 70 kg stands on a set of bathroom scales in a lift. Calculate the reading on the scales when the lift is accelerating downwards at 2 ms^{-2}.

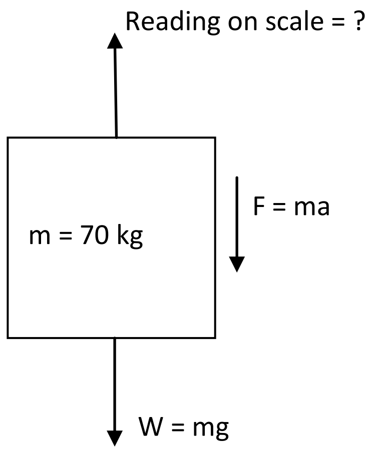
Solution

Remember that the reading on the scales = apparent weight = tension in the cable

a) Calculate F and W

F = m a
F = 70 x 2
F = 140 N

W = m g
W = 70 x 9.8
W = 686 N

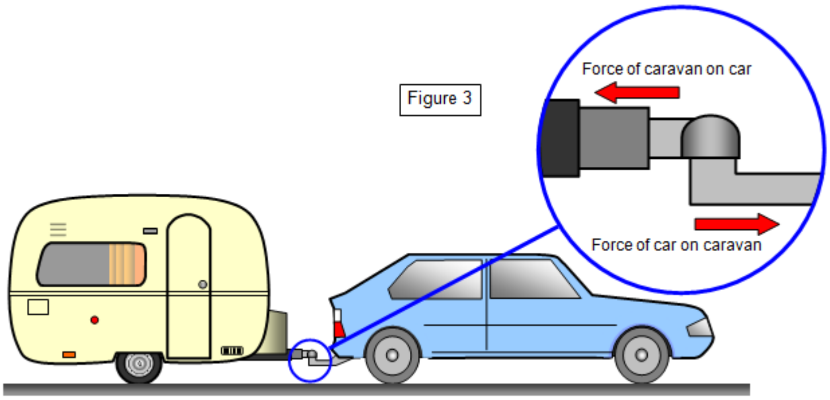


From the table on the previous page:
When the lift is **accelerating down**

F = W – reading
140 = 686 – reading
<u>Reading on scale = 546 N</u>

Internal Forces

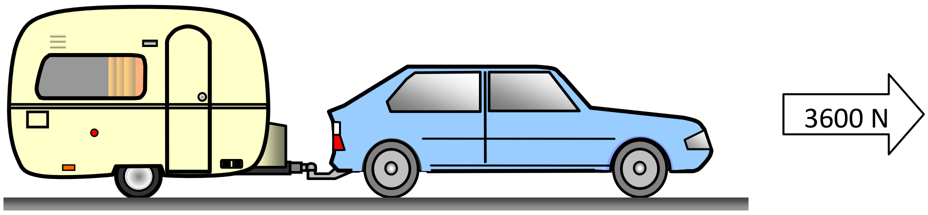
An example of an internal force is the tension in the towbar (magnified below) when a car is pulling a caravan.



In higher physics, a common question in the SQA exam you are asked is to calculate the tension between the two objects.

Example

A car of mass 700 kg pulls a 500 kg caravan with a constant engine thrust of 3.6 kN.
Calculate the tension in the towbar during the journey (ignoring friction)



Solution: **HINT Calculate the acceleration of the whole system using F = ma**

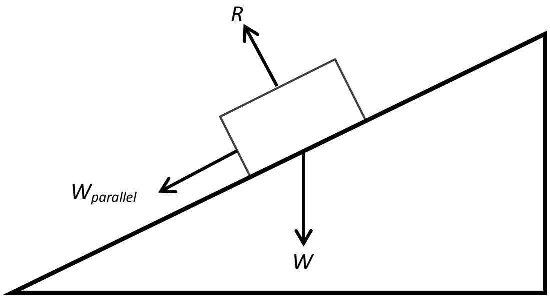
F = 3600 N	a = F / m
m = 500 + 700 = 1200 kg	a = 3600 / 1200
a = ?	a = 3 ms^{-2}

Use this acceleration to calculate the tension and use the mass of the caravan only as this is the mass of the object being pulled.

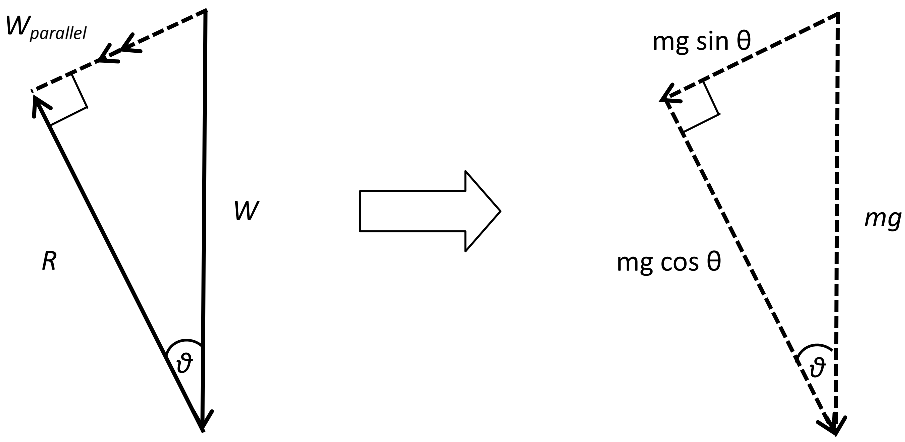
T = ?	T = ma
m = 500 kg	T = 500 x 3
a = 3ms^{-2}	T = 1500 N

Forces on a Slope

Ever wondered why a ball rolls down a hill without being pushed or a skier can ski down a run without an initial force. In order to understand why this happens we need to look at the forces exerted on an object resting on a slope



W is the weight of the object and R is the reaction force acting perpendicular to the slope.
If we draw these two forces tip to tail as described in section 1.1 we get the resultant force Wparallel shown in the diagram below.



$$\mathbf{W_{parallel} = mg\sin\theta}$$

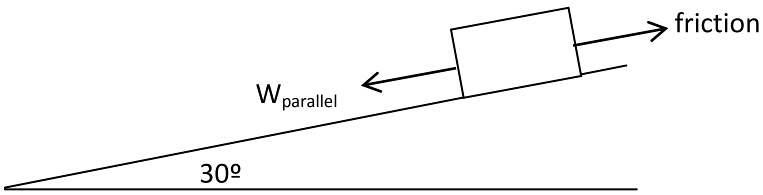
$$\mathbf{W_{perpindicular} = mg\cos\theta}$$

Forces on a Slope (continued)

Example

A car of mass 1000 kg is parked on a hill. The slope of the hill is 20^{o} to the horizontal. The brakes on the car fail. The car runs down the hill for a distance of 75 m until it crashes into a hedge. The average force of friction on the car as it runs down the hill is 250 N.

(a) Calculate the component of the weight acting down(parallel to) the slope.
(b) Find the acceleration of the car.
(c) Calculate the speed of the car just before it hits the hedge.



Solution

(a) $W_{parallel} = mg\sin\theta$
$= 1000 \times 9.8 \sin30$
$= 4900$ N

(b) $F = W_{parallel} - \text{Friction}$
$= 4900 - 250$
$= 4650$ N

$a = F / m$
$a = 4650 / 1000$
$a = 4.65\ ms^{-2}$

(c) s = 75 m
u = 0 (parked at rest)
v = ?
$a = 4.65\ ms^{-2}$
t =

$v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 4.65 \times 75$
$v = 26.4\ ms^{-1}$

Energy

Conservation of Energy

One of the fundamental principles of Physics is that of conservation of energy.

Energy cannot be created or destroyed, only converted from one form to another.

Work is done when converting from one form of energy to another. Power is a measure of the rate at which the energy is converted.

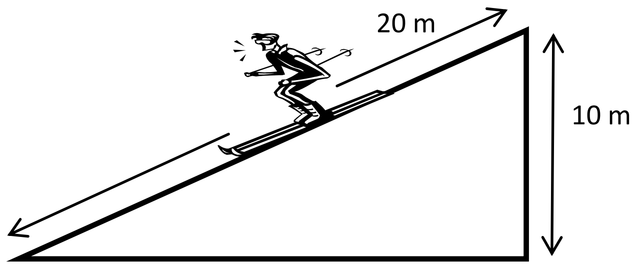
There are a number of equations for the different forms of energy:

$$\mathbf{E_w = Fs}$$
$$\mathbf{E_k = \tfrac{1}{2} mv^2}$$
$$\mathbf{E_p = mgh}$$
$$\mathbf{E_h = cm\Delta T}$$
$$\mathbf{E_h = ml}$$
$$\mathbf{E = Pt}$$

All forms of energy can be converted into any other form, so each of these equations can be equated to any other.

Example:

A skier of mass 60 kg slides from rest down a slope of length 20 m. The initial height of the skier was 10 m above the bottom and the final speed of the skier at the bottom of the ramp was 13 ms^{-1}.



Calculate:

(a) the work done against friction as the skier slides down the slope;

(b) the average force of friction acting on the skier.

Solution

(a) Calculate $\mathbf{E_p = mgh}$ to work out the amount of energy converted.

$E_p = mgh$

$E_p = 60 \times 9.8 \times 10$

$E_p = 5880$ J

(b) Use Ew = Fs

$5880 = F \times 20$

$F = 5880/20$

$F = 294$N

1.3 Collisions, Explosions and Impulse

Momentum

Conservation of Momenutum

Momentum is the measure of an object's motion and is the product of mass and velocity.

$$p = mv$$

Since velocity is a vector so is momentum, therefore momentum a direction and we must apply the convention of + ve and – ve directions.

An object can have a large momentum for two reasons, a large mass or a large velocity.

The law of conservation of linear momentum can be applied to the interaction (collision) of two objects moving in one dimension:

In the absence of net external forces total momentum before = total momentum after

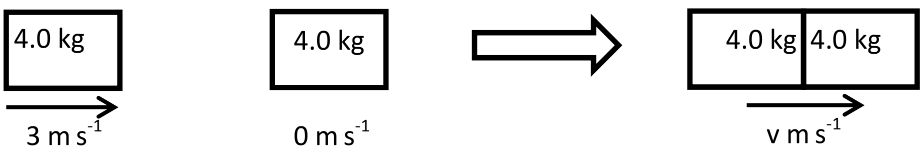
Collisions

The law of conservation of momentum can be used to analyse the motion of objects before and after a collision and an explosion. Let's deal with collisions first of all.

A collision is an event when two objects apply a force to each other for a relatively short time.

Example:
A trolley of mass 4.0 kg is travelling with a speed of 3 m s^{-1}. The trolley collides with a stationary trolley of equal mass and they move off together.
Calculate the velocity of the trolleys immediately after the collision.

Solution



$$\begin{aligned} m_1u_1 + m_2u_2 &= (m_1 + m_2)v \\ (4.0 \times 3) + (4.0 \times 0) &= (4 + 4) \times v \\ 12 &= 8v \\ v &= \frac{12}{8} \\ v &= 1.5 \text{ m s}^{-1} \end{aligned}$$

Kinetic Energy – Elastic and Inelastic Collisions

Elastic and Inelastic Collisions

When two objects collide their momentum is **always** conserved but, depending on the type of collision, their kinetic energy may or may not be. Take the two examples below:

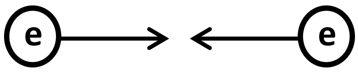
1.

If you were to witness this car crash you would hear it happen. There would also be heat energy at the point of contact between the cars.
These two forms of energy will have come from the kinetic energy of the cars, converted during the collision.
Here, kinetic energy is not conserved as it is lost to sound and heat. This is an **inelastic collision**.

In an **elastic collision**

E_k before $\neq$ **E_k after**

2.



When these two electrons collide they will not actually come into contact with each other, as their electrostatic repulsion will keep them apart while they interact.
There is no mechanism here to convert their kinetic energy into another form and so it is **conserved** throughout the collision.

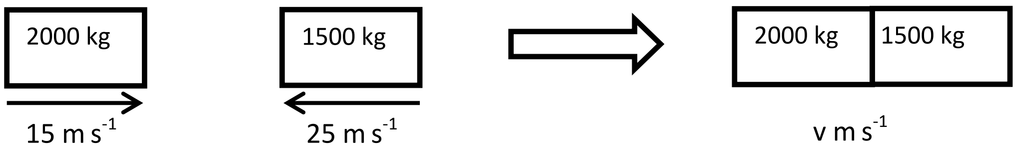
Elastic and Inelastic Collisions (continued)

Example:
A car of mass 2000 kg is travelling at 15 m s^{-1}. Another car, of mass 1500kg and travelling at 25 m s^{-1} collides with it head on. They lock together on impact and move off together.

(a) Determine the speed and direction of the cars after the impact.

(b) Is the collision elastic or inelastic? Justify your answer.

Solution



$$
\begin{array}{ccccc}
m_1u_1 & + & m_2u_2 & = & (m_1 + m_2)v \\
(\,2000 \times 15\,) & + & (1500 \times (-25)\,) & = & (\,2000 + 1500\,) \times v \\
3 \times 10^4 & + & (-\,37.5 \times 10^4\,) & = & 3500\,v \\
 & & v & = & \frac{-7.5 \times 10^3}{3500} \\
 & & & = & -\,\underline{2.1 \text{ m s}^{-1}}
\end{array}
$$

<u>The cars are travelling at 2.1 m s^{-1} to the left.</u>

(b)

$$
\begin{array}{lcccc}
E_k \text{ before} & = & \tfrac{1}{2}m_1u^2 & + & \tfrac{1}{2}m_2u^2 \\
 & = & (\,\tfrac{1}{2} \times 2000 \times 15^2\,) & + & (\tfrac{1}{2} \times 1500 \times 25^2\,) \\
 & = & 2.25 \times 10^5 & + & 4.69 \times 10^5 \\
 & = & 6.94 \times 10^5 \text{ J} & & \\
E_k \text{ after} & = & \tfrac{1}{2}m_{tot}v^2 & & \\
 & = & \tfrac{1}{2} \times 3500 \times 2.1^2 & & \\
 & = & 7.72 \text{x} 10^3 \text{ J} & &
\end{array}
$$

Explosions

In a simple explosion two objects start together at rest then move off in opposite directions. Momentum must still be conserved, as the total momentum before is zero, the total momentum after must also be zero.

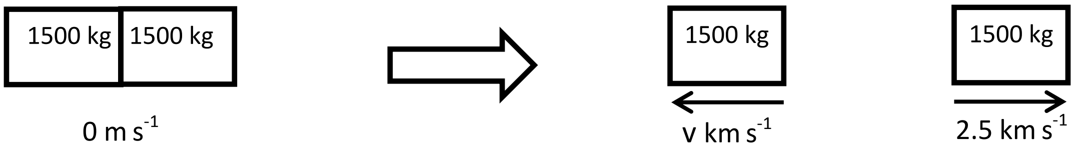
Example

An early Stark Jericho missile is launched vertically and when it reaches its maximum height it explodes into two individual warheads.

Both warheads have a mass of 1500 kg and one moves off horizontally, with a velocity of 2.5 km s^{-1} (Mach 9) at a bearing of 090°.

Calculate the velocity of the other warhead.

Solution



$$0 = m_1v_1 + m_2v_2$$

$$0 = 1500 \times v_1 + 1500 \times 2.5 \times 10^3$$

$$1500 \times v_1 = -1500 \times 2.5 \text{x} 10^3$$

$$v_1 = -\frac{3.75 \times 10^6}{1500}$$

$$v_1 = \underline{-2.5 \times 10^3 \text{ m s}^{-1}}$$

The negative sign in the answer indicates the direction of v_1 is opposite to that of v_2, i.e. 270° rather than 090°.

Newton’s Third Law and Momentum

Collisions

p_{total} before = p_{total} after

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$-m_1(v_1 - u_1) = m_2(v_2 - u_2)$$

$$\frac{-m_1(v_1 - u_1)}{t} = \frac{m_2(v_2 - u_2)}{t}$$

$$-m_1a_1 = m_2a_2$$

$$-F_1 = F_2$$

Collisions

p_{total} before = p_{total} after

$$0 = m_1v_1 + m_2v_2$$

$$m_1v_1 = -m_2v_2$$

$$\frac{m_1v_1}{t} = \frac{-m_2v_2}{t}$$

$$m_1a_1 = -m_2a_2$$

$$F_1 = -F_2$$

Impulse

Impulse

From Newton's Second Law:

$$F = ma = \frac{m(v-u)}{t}$$

$$F = ma = \frac{\cancel{(mv-mu)}}{t}$$

This expression states that the unbalanced force acting on an object is equal to the rate of change of momentum of the object. This was how Newton first stated his 2nd Second Law.

Impulse is the product of force and time, measured in N s. Impulse is the **cause** of a change in momentum. Rearranging Newton's Second Law from above:

$$Ft = mv - mu$$ (impulse has no symbol of its own)

$$Ft = \Delta p$$

Impulse is **equal** to the change in momentum, measured in kg m s^{-1}.

This means you can calculate the impulse from: $F \times t$ (N s), or $mv - mu$ (kg m s^{-1}).

A change in momentum depends on:

- The size of the force
- The time the force acts

Example:
A force of 100 N is applied to a ball of mass 150 g for a time of 0.020 s.
Calculate the final velocity of the ball.

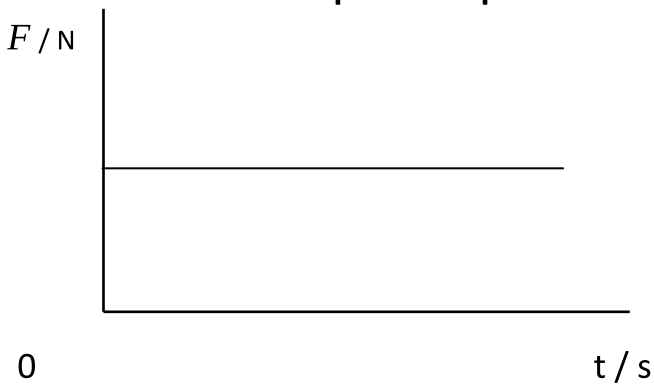
Solution

Solution:

F = 100N
m = 0.150 kg
t = 0.020 s
u = 0 m s^{-1}

$$Ft = mv - mu$$
$$100 \times 0.020 = 0.150 \times (v - 0)$$
$$2.0 = 0.150\, v$$
$$v = 2/0.15$$
$$v = 13.3 \text{ m s}^{-1}$$

Impulse Graphs



Ft = impulse = change in momentum

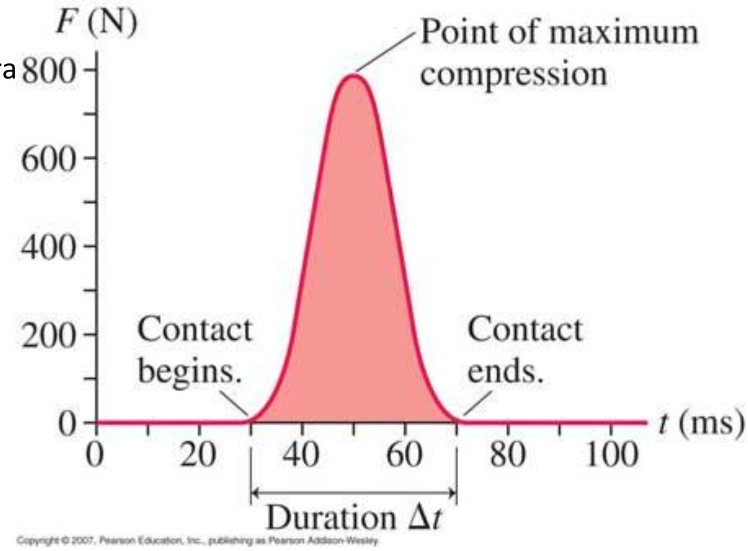
Ft = area under F - t graph

In reality, the force applied is not usually constant.
The analysis of the force acting on an object causing it to change speed can be complex. Often we will examine the force over time in graphical form.

Consider what happens when a ball is kicked.

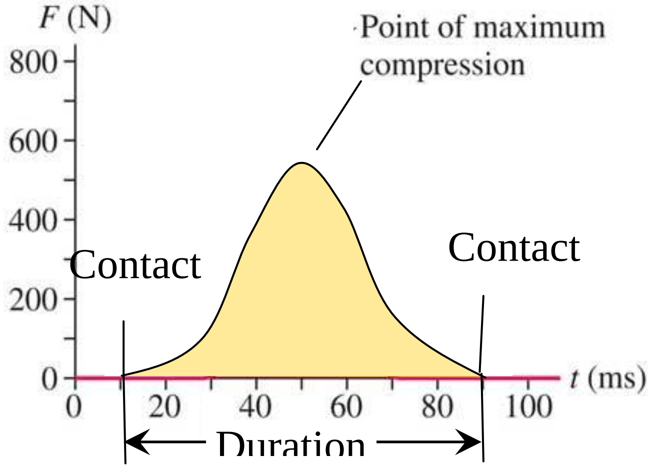
Once the foot makes contact with the ball a force is applied, the ball will compress as the force increases.
When the ball leaves the foot it will retain its original shape and the force applied will decrease.

This is shown in the gra



Impulse Graphs (continued)

If a ball of the same mass that is softer is kicked and moves of with the same speed as that above, then a graph such as the one below will be produced.

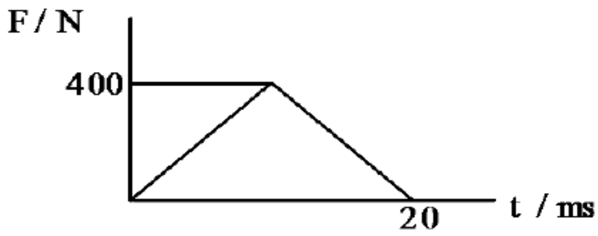


The maximum force applied is smaller but the time it is applied has increased. Since the ball has the same mass and moves off with the same speed its impulse will be the same as the original.

This results in a graph of the same area but different configuration.

Example

A tennis ball of mass 100 g, initially at rest, is hit by a racquet.
The racquet is in contact with the ball for 20 ms and the force of contact varies over this period, as shown in the graph.



Determine the speed of the ball as it leaves the racquet.

Solution

Impulse = area under graph

$= \frac{1}{2} \times 20 \times 10^{-3} \times 400$

$= 4$ N s

$u = 0$

$m = 100$ g $= 0.1$ kg

$v = ?$

$Ft = mv - mu$

$4 = 0.1v - (0.1 \times 0)$

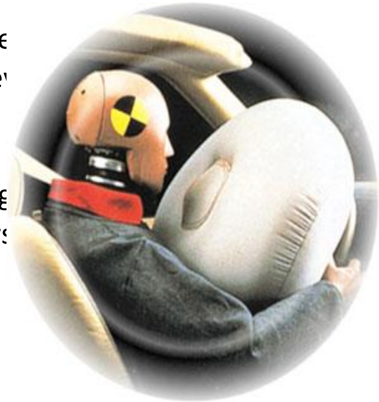
$4 = 0.1v$

$v = 40$ ms^{-1}

Practical Applications – Car Safety

Essentially the greater the time you can take to decelerate an object, the smaller the force you need to apply. If your face is slowed by the dashboard the time to stop after you make contact with the dashboard will be small, resulting in a large force and a big OWWWWWW!!

Airbags



The concept of the airbag - a soft pillow to land against in a crash - has be around for many years. The first patent on an inflatable crash-landing de during World War II. In the 1980s, the first commercial airbags appeared Stopping an object's momentum requires a force acting over a period of time. When a car crashes, the force required to stop an object is very larg because the car's momentum has changed instantly while the passengers has not, there is not much time to work with. The goal of any restraint system is to help stop the passenger while doing as little damage to him or her as possible. What an airbag wants to do is to slow the passenger's speed to zero with little or no damage. To do this it needs to **increase** the time over which the change in speed happens.

Crumple Zones

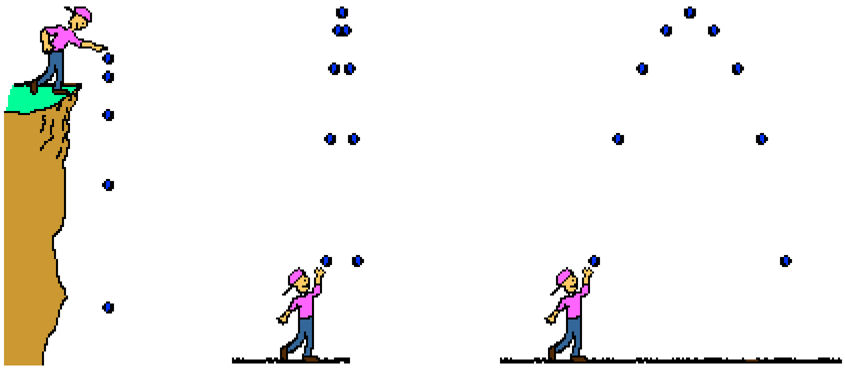
Placed at the front and the rear of the car, they absorb the crash energy developed during an impact. This is achieved by deformation. While certain parts of the car are designed to allow deformations, the passenger cabin is strengthened by using high-strength steel and more beams. Crumple zones delay the collision. Instead of having two rigid bodies instantaneously colliding, crumple zones **increase the time** before the vehicle comes to a halt. This **reduces the force** experienced by the driver and occupants on th or without a crumple zone.

1.4 Gravitation

Projectile Motion

Projectiles

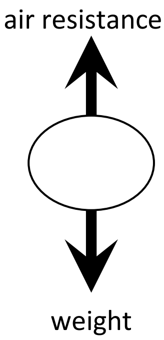
A projectile is any object, which, once projected, continues its motion by its own inertia and is influenced only by the downward force of gravity.



Most projectiles have both **horizontal and vertical** components of motion. As there is only a single force, gravity, acting in a single direction, which means only one of the components is being acted upon by the force. The two components are not undergoing the same kind of motion and must be treated **separately**.

Free Fall

When objects travel through the air, they have more than one force acting acting on them.
When an object is allowed to fall towards the Earth it will accelerate because of the force acting on it due to gravity, its **weight**. This will not be the only force acting on it though. There will be an upwards force due to **air resistance**.
Air resistance increases with speed; you may notice this if you increase your speed when cycling.

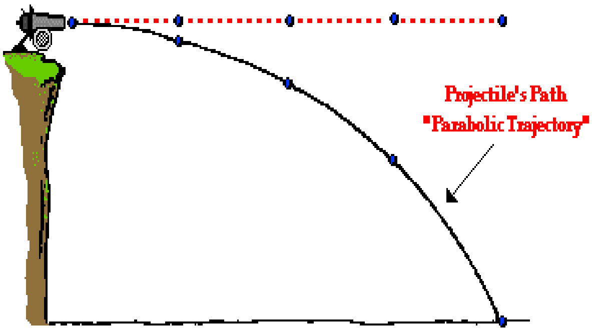


If an object is allowed to fall through a large enough distance then the air resistance force may increase to become the same magnitude as weight of the object. The forces are now balanced and the object will fall with constant velocity, known as **terminal velocity**.

Horizontal Projection

Here is a classic horizontal projectile scenario, from the time of Newton.
In projectile motion we **ignore all** air resistance, or any force other than gravity.

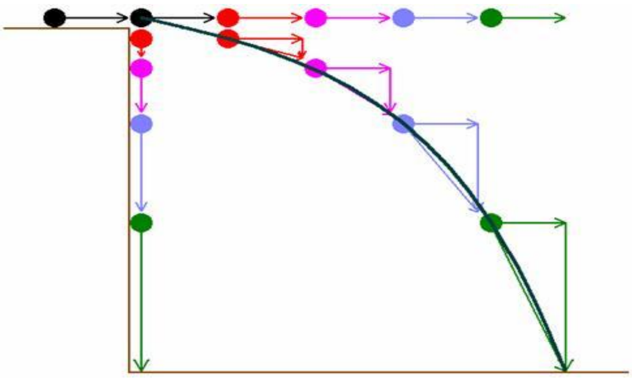
Analysis of this projectile shows the two different components of motion.



Horizontally: there are no forces acting on the cannonball and therefore the horizontal velocity is constant.

Vertically: The force due to gravity is constant in the vertical plane and so the cannonball undergoes constant acceleration.

The combination of these two motions causes the **curved** path of a projectile.



Example
The cannonball is projected horizontally from the cliff with a velocity of 100 m s^{-1}. The cliff is 20 m high. Determine:
(a) the vertical speed of the cannonball, just before it hits the water;
(b) if the cannonball will hit a ship that is 200 m from the base of the cliff.

Solution **(Hint: time is the only quantity which can cross the horizontal and vertical barrier. Calculate t on one side and use it on the other)**

Horizontal (use d = vt)	Vertical (use 3 equations of motion)
d = ?	s = 20 m
v = 100 m s^{-1}	u = 0
t = ?	v = ?
	a = 9·8 m s^{-2}
	t = ?

(a)
$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2 \times 9{\cdot}8 \times 20$
$v^2 = 392$
$v = 19{\cdot}8$ m s^{-1}

(b)
$v = u + at$
$19{\cdot}8 = 0 + (9{\cdot}8 \times t)$
$t = \dfrac{19{\cdot}8}{9{\cdot}8}$
$t = 2{\cdot}02$ s

Use t = 2.02 s on the horizontal side

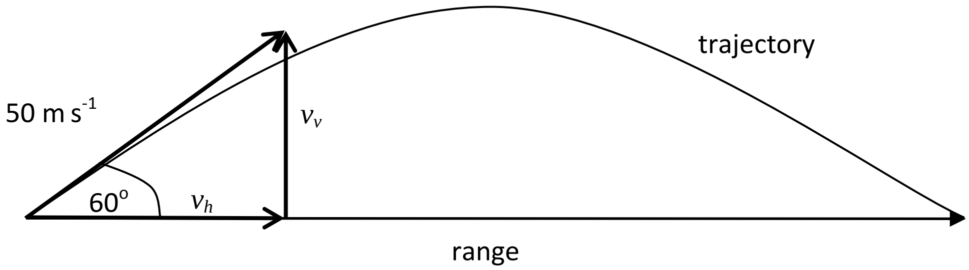
(b) ctd
$d = vt$
$d = 100 \times 2{\cdot}02$
$d = 202$ m

The cannonball will hit the ship.

Projection at an Angle

Projectiles at an angle are an application for our knowledge of splitting vectors into their horizontal and vertical components.

Example : The athlete has thrown the javelin at a velocity of 50 ms^{-1} at an angle of 60^{o} to the horizontal.



There is still only the single force of gravity acting on the projectile, so horizontal and vertical motions must still be treated very separately. This means that the velocity at an angle **must** be split into its **vertical and horizontal** components before any further consideration of the projectile.

You will **never** use the velocity at an angle (here 50 ms^{-1}) directly in any calculation!

Points to remember!!!
For projectiles fired at an angle above a horizontal surface:

1. The path of the projectile is symmetrical, in the horizontal plane, about the highest point. This means that:

$$\text{initial vertical velocity} = -\text{final vertical velocity}$$
$$u_v = -v_v$$

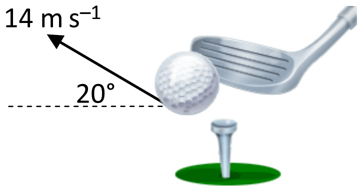
2. The time of flight = 2 × the time to highest point.

3. The vertical velocity at the highest point is zero.

Projection at an Angle Calculation

Example

A golfer hits a stationary ball and it leaves his club with a velocity of 14 ms^{-1} at an angle of 20° above the horizontal.



(a) Calculate:

 (i) the horizontal component of the velocity of the ball;

 (ii) the vertical component of the velocity of the ball.

(b) Calculate the time for the ball to reach its maximum height.

(c) Calculate the total time of flight of the ball

(d) How far down the fairway does the ball land?

Solution

Horizontal	Vertical
d = *?*	s = ?
$v_H = 13{\cdot}1 \text{ m s}^{-1}$	$u_v = 4{\cdot}8 \text{ m s}^{-1}$
t = *?*	v = 0 (at top)
	$a = -9{\cdot}8 \text{ m s}^{-2}$
	t = ?

(a) (i) $v_H = v \cos \Theta$

$v_H = 14 \cos 20$

$v_H = 13.1 \text{ ms}^{-1}$

(a) (ii) $v_H = v \sin \Theta$

$v_H = 14 \sin 20$

$v_H = 4.8 \text{ ms}^{-1}$

(b)

$$
\begin{aligned}
v &= u + at \\
0 &= 4{\cdot}8 + (-9{\cdot}8 \times t) \\
9{\cdot}8 \times t &= 4{\cdot}8 \\
t &= \frac{4{\cdot}8}{9{\cdot}8} \\
t &= 0{\cdot}49 \text{ s}
\end{aligned}
$$

(c) total time = 2 x 0.49 s

= 0.96 s

(d) $d = v_H t$

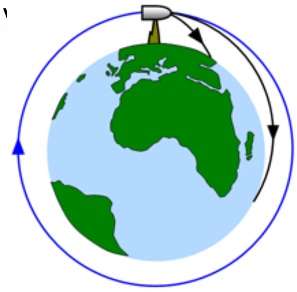
$d = 13.1 \times 0.96$

$d = 12.8 \text{ ms}^{-1}$

Newton and Gravity

Newton's Thought Experiment – Satellite's orbit as an Application of Projectiles

Isaac Newton, as well as giving us the three laws, came up with an ingenious thought experiment for satellite motion that predated the first artificial satellite by over 300 y



Essentially Newton suggested that if a cannon fired a cannonball it would fall towards the Earth. If it was fired at ever higher speeds then at some speed it would fall towards the Earth but never land since the curvature of the Earth would be the same as the flight path of the cannonball. This would then be a satellite. You would need a high mountain and an enormous cannon, but it would work.

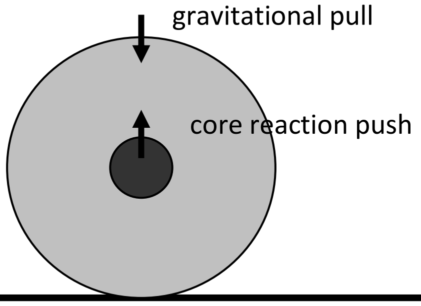
Gravity

There is much confusion in Physics of the difference between mass and weight. This could be because the word weight is used, wrongly, in everyday life. People are always talking about their 'weight'. The need to 'lose weight' or 'gain weight'.

What they are really talking about is their ***mass***. This is a measure of how much matter an object contains. This will only change if matter is added to or taken from the object. We can see this on a person as their body shape changes as matter is added or taken away.

What is Gravity?
Gravity is caused by mass. Any object that has mass will have its own gravitational field. The magnitude of the field depends on the mass of the object. Now, you are thinking that this is nonsense. If this were the case then everything on Earth would have it's own gravity and that's just not true. Well, actually, it is true. It's just that those gravitational forces are so small that we don't notice them. Remember, the gravitational pull of the Earth is 9.8 Nkg^{-1} and the Earth has a mass of 5.97×10^{24} kg just think how tiny the gravitational pull you exert must be.

Gravity is a force that permeates the entire universe; scientists believe that stars were formed by the gravitational attraction between hydrogen molecules in space. The attraction built up, over time, a large enough mass of gas such that the forces at the centre of the mass were big enough to cause the hydrogen molecules to fuse together, generating energy. This is what is happening in the centre of the sun. The energy radiating outwards from the centre of the sun counteracts the gravitational force trying to compress the sun inwards.



In time the hydrogen will be used up, the reaction will stop and the sun will collapse under its own gravity. If you expect to live for 4 or so billion years you could worry about this.

Newton's Universal Law of Gravitation

Newton produced what is known as the Universal Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

G is the universal constant of gravitation = $6{\cdot}67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$.

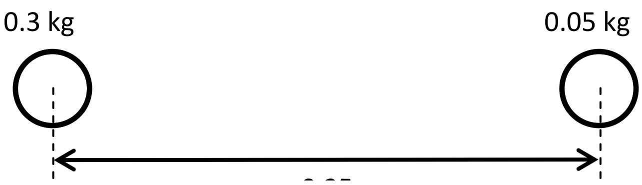
Newton's Law of gravitation states that the gravitational attraction between two objects (m_1 and m_2) is directly proportional to the mass of each object and is inversely proportional to the square of their distance (r) apart.

Gravitational force is always attractive, unlike electrostatic or magnetic forces.

The distance r between the two objects is the distance between their **centres of mass**. This is especially important when considering planetary bodies. For example, the radius of the orbit of the moon is only the distance from the surface of the Earth to the surface of the Moon, not the distance between their centres of mass.

Example
Consider a folder, of mass 0.3 kg and a pen, of mass 0.05 kg, sitting on a desk, 0.25 m apart.
Calculate the magnitude of the gravitational force between the two masses. (Assume they can be approximated to spherical objects).

Solution



$F = ?$
$m_1 = 0.3$ kg
$m_2 = 0.05$ kg
$r = 0.25$ m
$G = 6{\cdot}67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

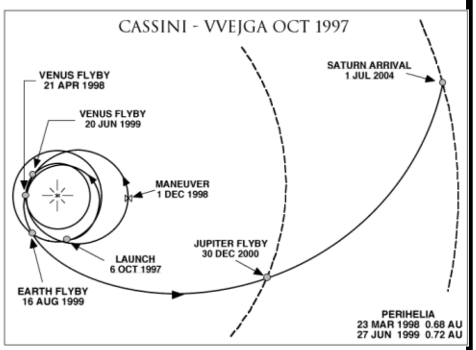
$$F = \frac{G\, m_1\, m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 0.3 \times 0.05}{0.25^2}$$

$$F = 1.6 \times 10^{-11}\ N$$

Application of Gravitational Force – The 'Slingshot Effect'

Another application of the gravitational force is the use made of the 'slingshot effect' by space agencies to get some 'free' energy to accelerate their spacecraft. Simply put they send the craft close to a planet, where it accelerates due to the gravitational field of the planet. Here's the clever part, if the trajectory is correct the craft then speeds past the planet with the increased speed. Don't get it right and you still get a spectacular crash into the planet, could be fun but a bit on the expensive side!

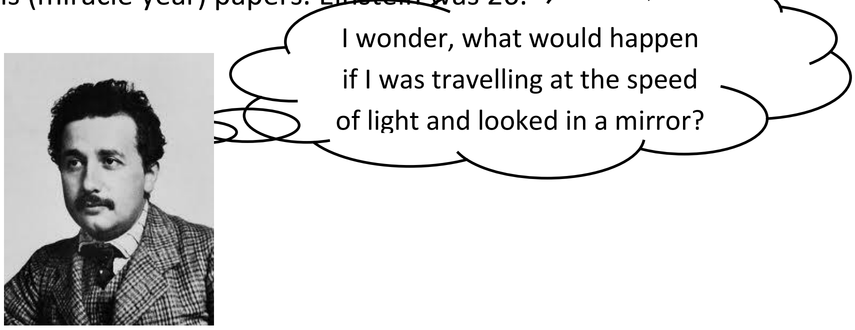


1.5 Special Relativity

Relativity

Introduction to Relativity

Einstein originally proposed his theory of special relativity in 1905 and it is often taken as the beginning of modern Physics. It was one of four world changing theories published by Einstein that year, known as the Annus Mirabilis (miracle year) papers. Einstein was 26.



Relativity has allowed us to examine the mechanics of the universe far beyond that of Newtonian mechanics, especially the more extreme phenomena such as black holes, dark matter and the expansion of the universe, where the usual laws of motion and gravity appear to break down.
Special Relativity was the first theory of relativity Einstein proposed. It was termed as ‘special’ as it only considers the ‘special’ case of reference frames moving at **constant** speed. Later he developed the theory of general relativity which considers accelerating frames of reference.

Reference Frames

Relativity is all about observing events and measuring physical quantities, such as distance and time, from different reference frames. Here is an example of the same event seen by three different observers, each in their own frame of reference:

Event 1: You are reading your Kindle on the train. The train is travelling at 60 mph.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 60 mph
3	Passenger on train travelling at 60 mph in opposite direction	You are travelling towards them at 120 mph.

This example works well as it only involves objects travelling at relatively low speeds. The comparison between reference frames does not work in quite the same way, however, if objects are moving close to the speed of light.

Event 2: You are reading your Kindle on an interstellar train. The train is travelling at 2×10^8 m s^{-1}.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 2×10^8 ms^{-1}
3	Passenger on train travelling at 2×10^8 m $^{-1}$ in opposite direction	You are travelling towards them at **4×10^8 ms^{-1}**

The observation made by observer 3 is impossible as an object cannot travel faster than the speed of light in any reference frame and it would certainly be impossible to watch something travel faster than light, so this scenario is impossible.

The Principles of Relativity

The Principles of Relativity - Introduction

Using his imagination and performing thought experiments (gedanken) like those above, Einstein came up with two principles, or postulates, to explain the problem of fast moving reference frames. These were later proved with a vast array of data from many different experiments and became very clear once we started communicating with satellites, in orbit.

The postulates of Special Relativity:

1. **When two observers are moving at constant speeds relative to one another, they will observe the same laws of physics.**
2. **The speed of light (in a vacuum) is the same for all observers.**

This means that no matter how fast you go, you can never catch up with a beam of light, since it always travels at $3{\cdot}0 \times 10^8$ ms^{-1} **relative** *to you*.

If you (or anything made of matter) were able to travel as fast as light, light would still move away or towards you at $3{\cdot}0 \times 10^8$ ms^{-1}, as you are stationary in your own reference frame.

The most well-known experimental proof is the Michelson-Morley interferometer experiment. Maxwell's electromagnetism equations also corroborated these postulates.

Example: If a car ship is travelling through space at 90% of the speed of light and then switches on its headlights. The passenger of the car will see the beams of the headlights travel away from them at 3×10^8 ms^{-1}.
An observer on Earth will also observe light of the beams travelling at 3×10^8 ms^{-1}.

The speed of light, **c**, is constant in and between all reference frames and for all observers.
These principles have strange consequences for the measurement of distance and time between reference frames.

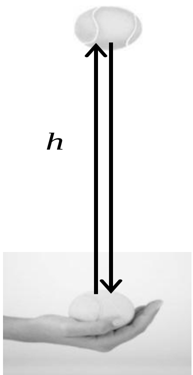
Time Dilation

Time Dilation

We can conduct a thought experiment of our own, showing that one consequence of the speed of light being the same for all observers is that time experienced by all observers is not necessarily the same. There is no universal clock that we can all refer to – we can only make measurements of time as we experience it.

Time is different for observers in different reference frames because the path they observe for a moving object is different.

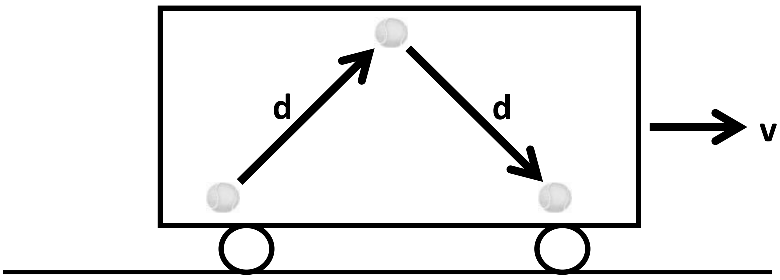
Event 1: Inside a moving train carriage, a tennis ball is thrown straight up and caught in the same hand.



Observer 1, standing in train carriage, throws tennis ball straight up and catches it in the same hand.
In Observer 1's reference frame they are stationary and the ball has gone straight up and down.
Observer 1 sees the ball travel a total distance of **2h**.
The ball is travelling at a speed **v**.
The period of time for the ball to return to the observers hand is:

$$t = 2h / v$$

Observer 2, standing on the platform watches the train go past at a speed, **v,** and sees the passenger throw the ball. However, to them, the passenger is also travelling horizontally, at speed **v**. This means that, to Observer 2, the tennis ball has travelled a horizontal distance, as well as a vertical one.



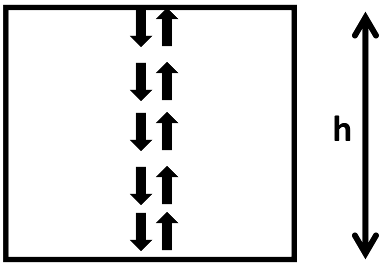
Observer 2 sees the ball travel a total distance of **2d.**
The period of time for the ball to return to the observers hand is:

$$t' = 2d / v$$

For observer 2, the ball has travelled a greater distance, in the same time.

Time Dilation (continued)

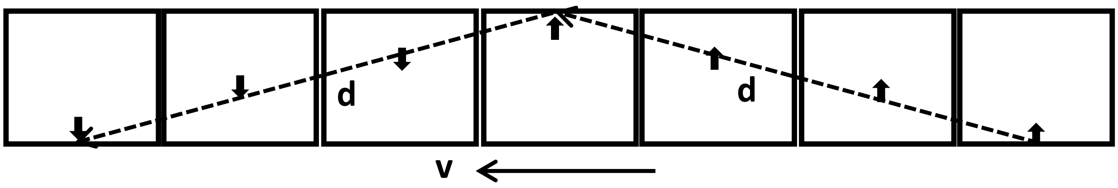
Event 2: You are in a spaceship travelling to the left, at speed **v**. Inside the spaceship cabin, a pulsed laser beam is pointed vertically up at the ceiling and is reflected back down. The laser emits another pulse when the reflected pulse is detected by a photodiode.



Reference frame 1: you,inside the cabin.
The beam goes straight up, reflects of the ceiling and travels straight down.

Period of pulse **$t = 2h / c$**

Reference frame 2: Observer on another, stationary ship.



Period of pulse **$t' = 2d / c$**

The time for the experiment as observed by the stationary ship ,**t'**, is greater than the time observed by you when moving with the photodiode **t**, i.e. what you might observe as taking 1 second could appear to take 2 seconds to your stationary colleague. Note that you would be unaware of any difference until you were able to meet up with your colleague again and compare your data.

Time Dilation Equation and the Lorentz Factor

$$\mathbf{t}' = \frac{\mathbf{t}}{\sqrt{\mathbf{1} - \left(\frac{\mathbf{v^2}}{\mathbf{c^2}}\right)}}$$

t' is always observed by the <u>stationary</u> observer, observing the object moving at speed. E.g. the person on a train platform watching the train go by, or an observer on Earth watching a fast moving ship.

Note this is often written as: $\mathbf{t' = t\gamma}$

where **γ** is known as the Lorentz Factor. It is used often in the study of special relativity and is given by:

$$\gamma = \frac{\mathbf{1}}{\sqrt{\mathbf{1} - \left(\frac{\mathbf{v^2}}{\mathbf{c^2}}\right)}}$$

<u>Example:</u>
A rocket is travelling past Earth at a constant speed of $2.7 \times 10^8\ ms^{-1}$.
The pilot measures the journey as taking 240 minutes.
How long did the journey take when measured by an observer on Earth?

<u>Solution:</u>

t = 240 minutes
$c = 3 \times 10^8\ ms^{-1}$
$v = 2.7 \times 10^8\ ms^{-1}$
t' = ?

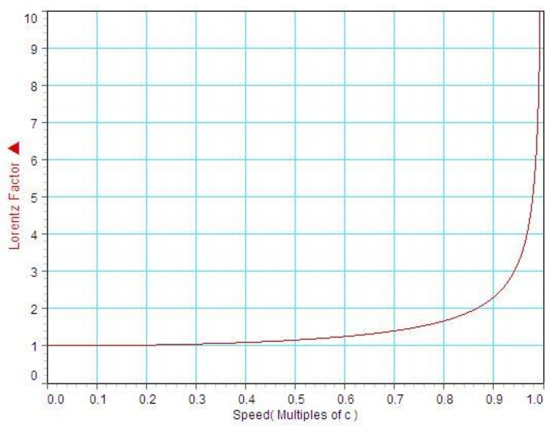
$$t' = \frac{t}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$t' = \frac{240}{\sqrt{1 - \left(\frac{2.7 \times 10^8}{3.0 \times 10^8}\right)^2}}$$

t' = 550 minutes

An observer on Earth would measure the journey as taking 550 minutes.

Why we do not notice relativistic time differences in everyday life?
A graph of the Lorentz factor versus speed (measured as a multiple of the speed of light) is shown below.



We can see that for small speeds (i.e. less than 0.1 times the speed of light) the Lorentz factor is approximately 1 and relativistic effects are negligibly small. Even 0.1 times the speed of light is 300,000 ms^{-1} or 1,080,000 km h^{-1} or about 675,000 mph – a tremendously fast speed compared to everyday life.

However, the speed of satellites is fast enough that even these small changes will add up over time and seriously affect the synchronisation of global positioning systems (GPS) and television satellites with users on the Earth. They have to be specially programmed to adapt for the effects of special relativity (and also general relativity, which is not covered here). Very precise measurements of these small changes in time have been performed

Application of Time Dilation

Further evidence in support of special relativity comes from the field of particle physics, in the form of the detection of a particle called a muon at the surface of the Earth. Muons are produced in the upper layers of the atmosphere by cosmic rays (high-energy protons from space). The speed of muons high in the atmosphere is 99·9653% of the speed of light.

The half-life of muons when measured in a laboratory is about 2·2 μs.

Example: Show, by calculation, why time dilation is necessary to explain the observation of muons at the surface of the Earth.

Solution:

$t = 2.2$ □s $= 2.2 \times 10^{-6}$ s
$v = 0.999653 \times 3.00 \times 10^{8} = 2.998956 \times 10^{8}$ ms^{-1}
$d = ?$

$d = vt$
$d = 2.998956 \times 10^{8} \times 2.2 \times 10^{-6}$
$d = 660$ m

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

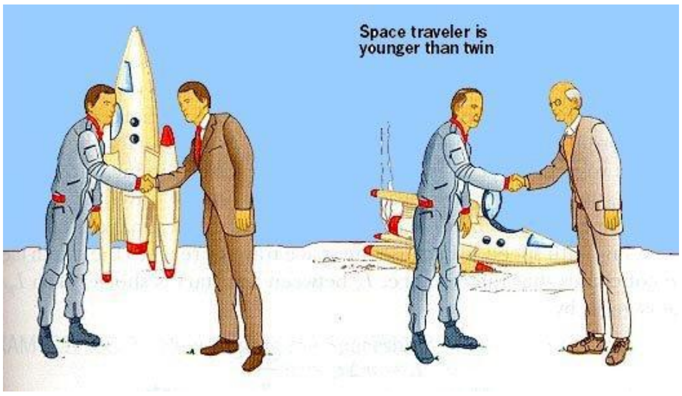
$$t' = \frac{2.2 \times 10^{-6}}{\sqrt{1 - (0.999653)^2}}$$

$t' = 84$ s
$d' = vt$
$d' = 2.998956 \times 10^{8} \times 84 \times 10^{-6}$
$d' = 2.52 \times 10^{4}$ m

In the reference frame of an observer on Earth the half-life of the muon is recorded as 84 μs and therefore from this perspective, the muon has enough time to travel the many kilometres to the Earth's surface.

A Twin Paradox

You leave Earth and your twin to go on a mission in a spaceship travelling at 90% the speed of light on a return journey that lasts 20 years. When you get back you find that 46 years will have elapsed on Earth. Your clock will have run slowly compared to one on Earth, however as far as you were concerned the clock would have been working correctly on your spaceship.



You will look 26 years younger than your twin.

Length Contraction

Another implication of Einstein's theory is the shortening of length when an object is moving. Consider the muons discussed above. Their large speed means they experience a longer half-life due to time dilation. An equivalent way of thinking about this is that the fast moving muons observe a much shorter (or contracted) distance travelled, by the same amount as the time has increased (or dilated). A symmetrical formula for length contraction can be derived.

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

Where **l** is the distance measured by an observer who is stationary and **l'** the distance observed by the observer who is moving at speed.

<u>Example</u>
Let's take the example of a space ship flying away from Earth towards Proxima Centauri, our nearest star, to study the observations due to length contraction. The distance to Proxima Centauri is 4.2 ly. Length contraction only takes place in the direction that the object is travelling. For the pilot of the space ship, this means that they will measure the distance, in front of them, between Earth and Proxima as less than the distance measured by a stationary observer.

Let's say the spaceship is travelling at 0.8 c.

l = 4.2 ly
l' = ?
v = 0.8 c

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$l' = 4.2\sqrt{1 - 0.8^2}$$

$$l' = 2.52$$

$$l' = 2.5 \text{ ly}$$

So the Pilot of the ship measures their journey as 2.5 ly.

Length Contraction – A Paradox

There is an apparent paradox thrown up by special relativity: consider a train that is just longer than a tunnel. If the train travels at high speed through the tunnel does length contraction mean that, from our stationary perspective, it fits inside the tunnel? How can this be reconciled with the fact that from the train's reference frame the tunnel appears even shorter as it moves towards the train? The key to this question is simultaneity, i.e. whether different reference frames can agree on the exact time of particular events. In order for the train to fit in the tunnel the front of the train must be inside at the same time as the back of the train. Due to time dilation, the stationary observer (you) and a moving observer on the train cannot agree on when the front of the train reaches the far end of the tunnel or the rear of the train reaches the entrance of the tunnel. If you work out the equations carefully then you can show that even when the train is contracted, the front of the train and the back of the train will not both be inside the tunnel at the same time!

1.6 The Expanding Universe

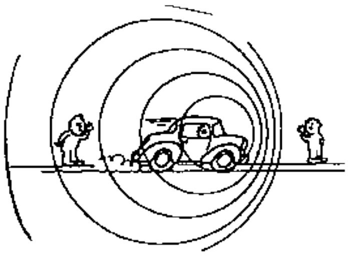
The Doppler Effect and Redshift

The Doppler Effect

The Doppler Effect is the change in the observed frequency of a wave, when the source or observer is moving.

In this course we will concentrate on a wave source moving at **constant speed** relative to a stationary observer.

You have already experienced the Doppler Effect many times. The most noticeable is when a police car, ambulance or fire engine passes you. You hear the pitch of their siren increase as they come towards you and then decrease as they move away. Another memorable example is the sound of a very fast moving vehicle, such as a Formula 1 car passing you (or passing a microphone on the television), the sound of the engine rises and falls in frequency as it approaches, passes and moves away.



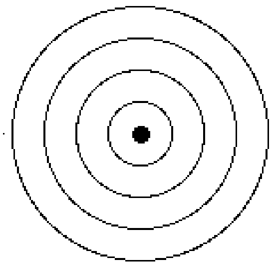
The Doppler Effect applies to all waves, including light.

Uses of the Doppler Effect

- Police radar guns use the Doppler effect to measure the speed of motorists.
- Doppler is used to measure the speed of blood flow in veins to check for deep vein thrombosis [DVT] in medicine.

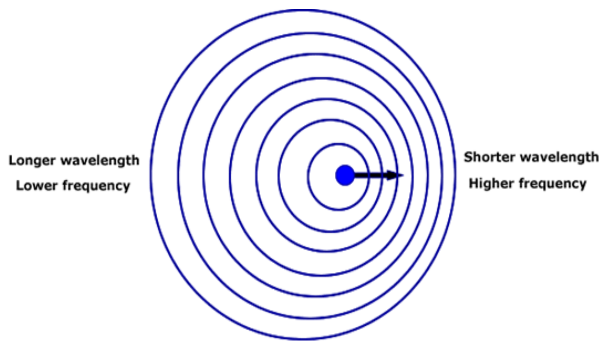
Stationary Source

A stationary sound source produces sound waves at a constant frequency **f**, and the wavefronts propagate symmetrically away from the source at a constant speed, which is the speed of sound in the medium. The distance between wave-fronts is the wavelength. All observers will hear the same frequency, which will be equal to the actual frequency of the source: $\mathbf{f = f_0}$.



Moving Source

The sound source now moves to the right with a speed $\mathbf{v_s}$. The wavefronts are produced with the same frequency as before, therefore the period of each wave is the same as before. However, in the time taken for the production of each new wave the source has moved some distance to the right. This means that the wavefronts on the left are created further apart and the wavefronts on the right are created closer together. This leads to the spreading out and bunching up of waves you can see to the right and hence the change in frequency.



The frequency of the source will remain constant, it is the observed frequency that changes.

The Doppler Effect Equations

More relevant to our learning in this section, the Doppler Effect is highly prominent in our observations of the universe and provides some of the strongest evidence for major theories such as the Big Bang and an expanding universe.

For a stationary observer with a wave source **moving towards them**, the relationship between the frequency, $\mathbf{f_s}$, of the source and the observed frequency, $\mathbf{f_o}$, is:

$$\mathbf{f_o = f_s\left(\frac{v}{v - v_s}\right)}$$

v = speed of the wave
v_s = speed of source
f_s = frequency source
f_o = observed frequency

For a stationary observer with a wave source **moving away from them**, the relationship between the frequency, $\mathbf{f_s}$, of the source and the observed frequency, $\mathbf{f_o}$, is:

$$\mathbf{f_o = f_s\left(\frac{v}{v + v_s}\right)}$$

This second scenario is exactly what is observed when we look at the light from distant stars, galaxies and supernovae, evidence that the universe is expanding. These relationships also allow us to calculate the speed at which an exoplanet is orbiting its parent star, or the velocity of stars orbiting a galactic core, which has lead us to theorise the existence of dark matter.

An Example of the Doppler Effect - Red Shift

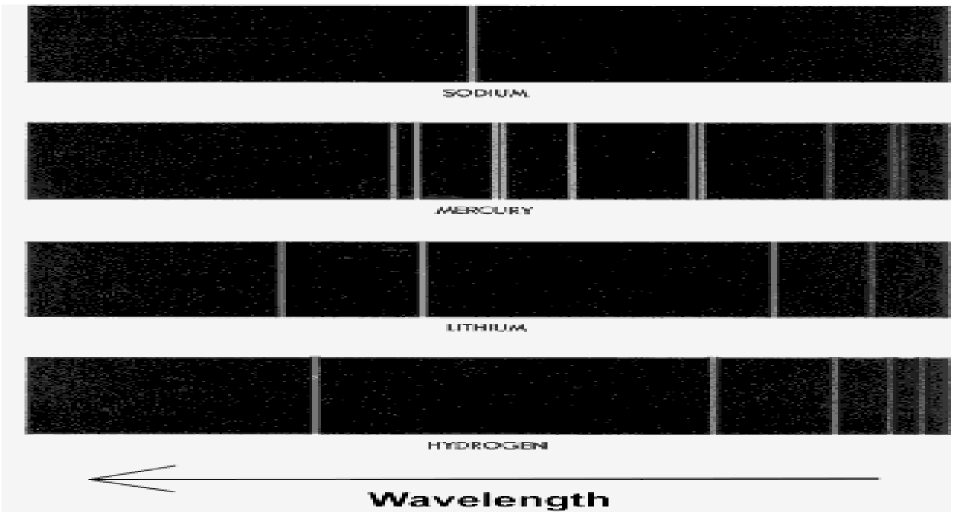
Redshift is an example of the Doppler Effect. The light from stars, as observed from Earth, is always reduced in frequency and shifted towards the red (longer wavelengths) end of the spectrum. This is because the stars and galaxies are sources of light which are moving away from us.

Redshift has always been present in the light reaching us from stars and galaxies but it was first noticed by astronomer Edwin Hubble, in the 1920's, when he observed that the light from distant galaxies was shifted to the red end of the spectrum (longer wavelengths).

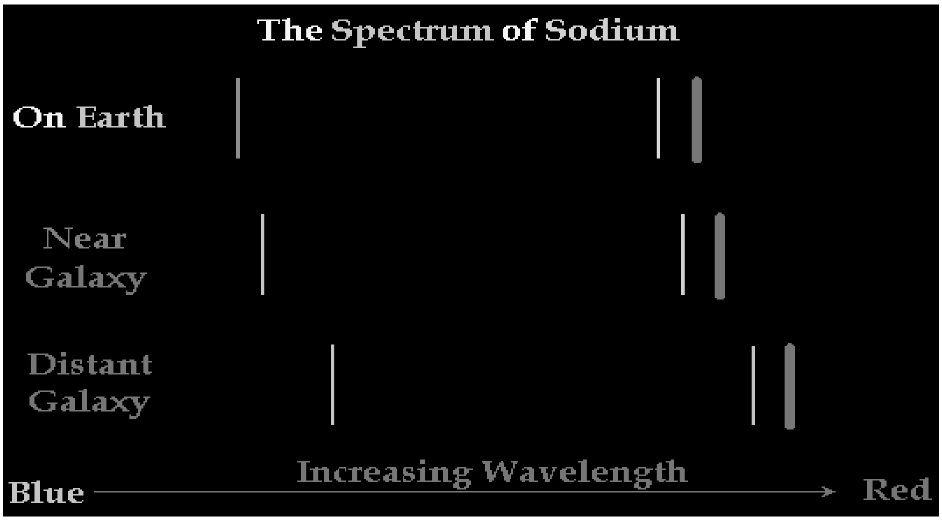
The light emitted by a star is made up of the line spectra emitted by the different elements present in that star. Each of these line spectra is an identifying signature for an element and these spectra are constant throughout the universe. You will learn a lot more about spectra in the Particles and Waves unit of this course.

Since these line spectra are so recognisable, we can compare the spectra produced by these elements, on Earth, with the spectra emitted by a distant star or galaxy.

Examples of line spectra of different elements



Hubble examined the spectral lines from various elements and found that the spectra emitted by each galaxy were shifted **towards the red** by a specific amount. This **shift** was due to the **galaxy moving away from the Earth** at speed, causing the Doppler Effect to be observed. The **bigger the magnitude of the shift the faster the galaxy** was moving.



Redshift of a Galaxy Equation

Redshift, **z**, of a galaxy is given by:

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$$

Redshift of galaxies, travelling at non-relativistic speeds, can also be shown to be the ratio of the velocity of the galaxy to the velocity of light:

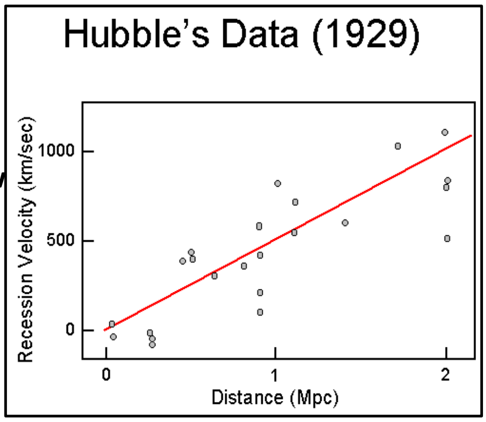
$$z = \frac{v_{galaxy}}{c}$$

As redshift is always calculated from the ratio of quantities with the same unit, it has **no unit of its own**.

Over the course of a few years Hubble examined the red shift of galaxies at varying distances from the Earth. He found that the further away a galaxy was the faster it was travelling away from us. The relationship between distance and speed of a galaxy is known as Hubble's Law.

Hubble's Law

The graph below shows the data collected by Hubble.
It shows the relationship between the velocity of a galaxy **v**, as it recedes from us, and its distance **d**, known as **Hubble's Law**



The gradient of the line is known as H_o – Hubble's constant.

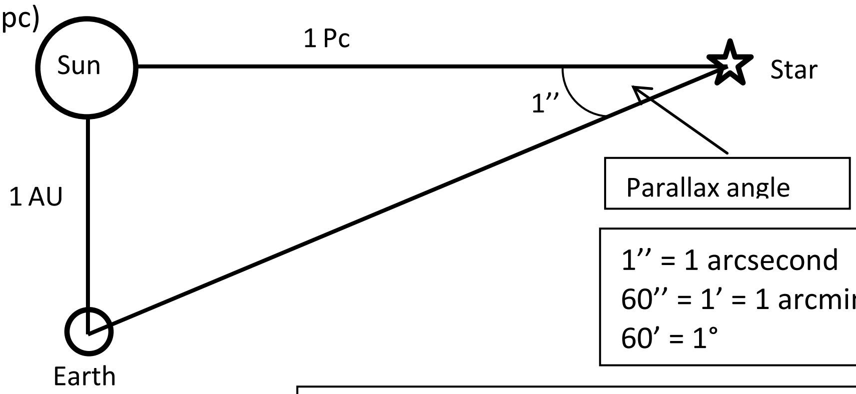
$$H_o = v / d$$

or

$$v = H_o d$$

Understanding Hubble's Law

The parsec (pc)



1'' = 1 arcsecond
60'' = 1' = 1 arcminute
60' = 1°

1 pc is the distance between the sun and an astronomical object (star, galaxy etc.) when the parallax angle is 1''.

The value of the Hubble constant is not known exactly, as the exact gradient of the line of best fit is subject to much debate. However, as more accurate measurements are made, especially for the distances to observable galaxies, the range of possible values has reduced. It is currently thought to lie between 50 – 80 $kms^{-1}\ Mpc^{-1}$, with the most recent data putting it at 70.4 ± 1.4 $kms^{-1}\ Mpc^{-1}$.

Calculating the Age of the Universe

Hubble's observations show that galaxies are moving away from the Earth and each other in all directions, which suggests that the universe is expanding. This means that in the past the galaxies were closer to each other than they are today. By working back in time it is possible to calculate a time when all the galaxies were at the same point in space. This allows the age of the universe to be calculated.

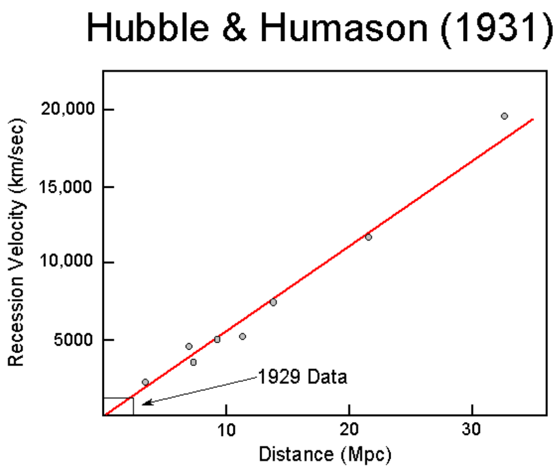
v = speed of galaxy receding from us
d = distance of galaxy from us
H_0 = Hubble's constant
t = time taken for galaxy to travel that distance, **i.e. the age of the universe**

$$t = \frac{d}{v} \qquad (v = H_0 d)$$

$$t = \frac{d}{H_0 d}$$

$$t = \frac{1}{H_0}$$

Currently, using this method, NASA estimate the age of the universe to be **13·7 billion years**.
Since Hubble's time, there have been other major breakthroughs in astronomy and our ability to make accurate observations of very distant objects. All of these support the findings of Hubble, but allow the age of the universe to be calculated even more accurately.



Evidence for the Expanding Universe

It is generally accepted, based on the evidence given previously, that the universe is expanding. What is not known however is, what is going to happen to the universe in the future? There are essentially two scenarios.
1. Closed universe: the universe will slow its expansion and eventually begin to contract.
2. Open universe: the universe will continue to expand forever.

Which of the two scenarios is more likely depends on one factor, what is the mass of the universe? We come back to our old friend gravity.

How can we measure the mass of objects in space? You would need a big set of scales.
In fact astronomers can relate the orbital speed of galaxies to their masses.
The problem is that the masses measured seem to be bigger than the mass that can be accounted for by the number of stars present in a galaxy.
This leads to the theory of '**Dark Matter'**. Basically there appears to be stuff there that we can't see and don't know what it is, so for the moment give it a name and hope we find out what it actually is later.

Now if that was the only thing it might not be too bad, but the universe is expanding at a greater rate than astronomers would expect. It seems that something appears to be opposing the gravitational force.

"Give it a name" I hear you cry!

So they did, they called it **Dark Energy**. Who said astronomers aren't creative thinkers?

This increased expansion appears to verify Einstein's inclusion of the cosmological constant in his Special Theory of Relativity. This was an inclusion that he called "His greatest blunder." He only added it because he could not agree that the universe would eventually collapse, a theory around at that time. There was no evidence to support his theory at the time and unfortunately he died before the evidence that could support his theory was uncovered.

Big Bang Theory

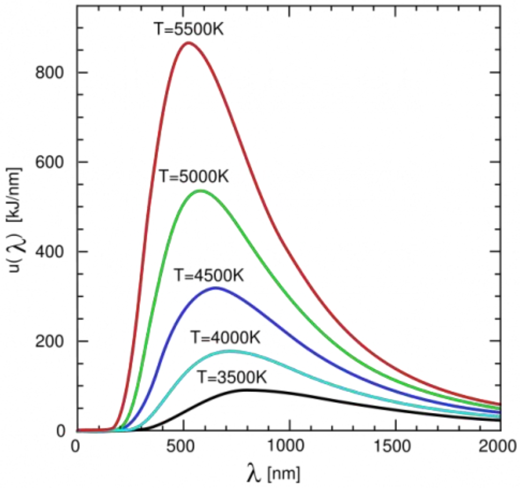
The universe started with a sudden appearance of energy which consequently became matter and is now everything around us. There were two theories regarding the universe

- The Steady State Universe: where the universe had always been and would always continue to be in existence.
- The Created Universe: where at some time in the past the universe was created.

Ironically the term 'Big Bang' was coined by Fred Hoyle a British astronomer who was the leading supporter of the Steady State theory and who was vehemently opposed to the, currently named, Big Bang theory.

What was the evidence that finally swung the balance towards the Big Bang theory?

We first need to consider how it is possible to determine the temperature of distant stars and galaxies. You will have seen what happens to a piece of iron as it is heated, as it gets hotter its colour changes from dull red to bright red to orange then yellow.



The observant amongst you may realize that these are the first colours in the visible spectrum. The temperature of an object determines the frequency of light it emits. This idea has been with us for a long time; Jožef Stefan proposed in 1879 that the power irradiated from an object was proportional to its temperature in Kelvin to the fourth power.

$$P = \sigma T^4$$

Where Stefan's constant is $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$.

What this means is that by examining the spectrum of a distant star, its temperature can effectively be determined. This graph also shows that intensity increases with temperature.

Gamow, Alpher and Herman, three physicists, had produced a paper in 1948 that if the Big Bang had actually taken place then there would be a residual background EM radiation, in the microwave region, in every direction in the sky representing a temperature of around 2.7K.

This value for the wavelength of the light and it's consequent equivalent temperature was arrived at by considering how the light produced at the Big Bang would have changed as the universe expanded.

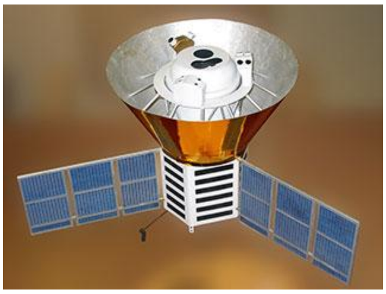
The discovery of this background radiation was another example of scientists finding something they weren't looking for.

Arnold Penzias and Robert Wilson were working for Bell Labs in the USA. They were working with a special radio telescope [shown in the picture above] experimenting with satellite communication.

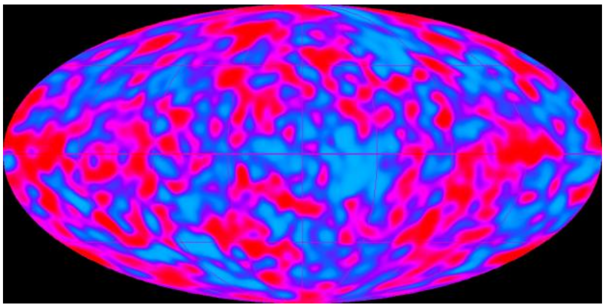
They were getting a residual signal that seemed to come from outside the galaxy. At first they though it was actually due to pigeon droppings from the pigeons that roosted in the horn. Finally they realised that they had found the echo of the Big Bang.

Big Bang Theory (continued)

In 1989 a satellite was launched to study the background radiation, it was called the Cosmic Background Explorer [COBE].
In 1992 it was announced that COBE had managed to measure fluctuations in the background radiation. This was further evidence to support the Big Bang theory.



An image of the fluctuations is shown below.

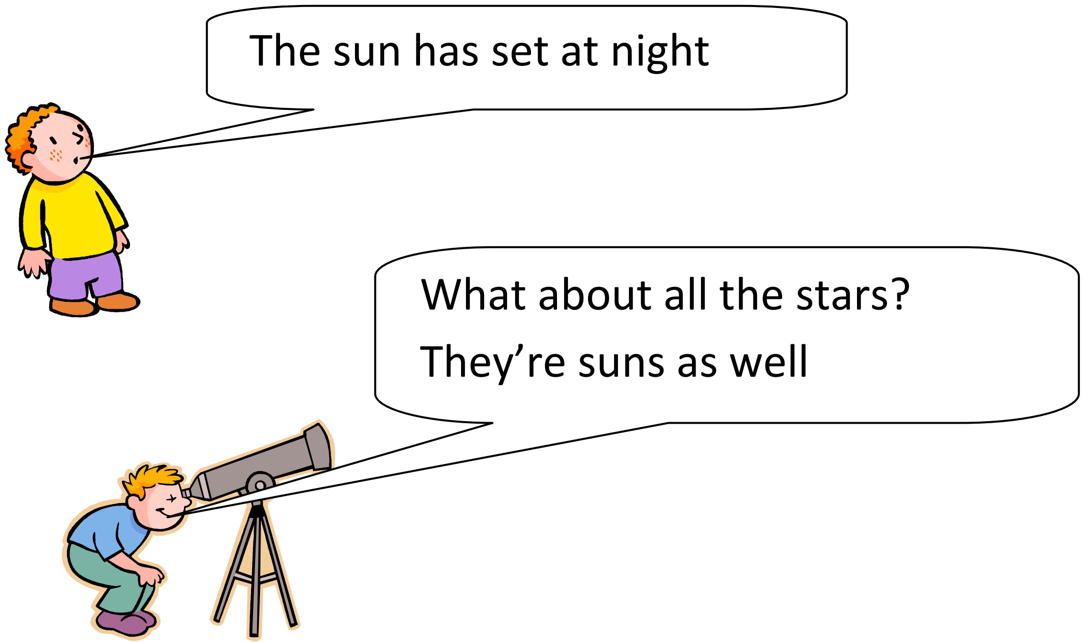


Other evidence to support the Big Bang theory includes the relative abundances of hydrogen and helium in the universe.
Scientists predicted that there should be a significantly greater proportion of hydrogen in the universe. The next most abundant should be helium.
The elements present in the universe can be determined by spectroscopy, which you will study later in unit 3.
The latest proportions are given in the table shown. These observations conform to the predicted proportions.

Element	**Relative Abundance**
Hydrogen	10 000
Helium	1 000
Oxygen	6
Carbon	1
All others	1

Olber's Paradox - Big Bang Theory (continued)

Another is the explanation for Olber's paradox. His paradox was in answer to the question, "why is the sky dark at night?"
This is not as obvious as you first might imagine.



Wullie replies:

If the universe followed the Steady State model then there should be an even distribution of stars in all directions. All the stars in the universe should be visible. This means the light from the stars should reach Earth and the sky should be bright.

The Big Bang theory gives a finite age to the universe, and only stars within the observable universe can be seen. This means that only stars within the distance of 15 000 light years will be observed. Not all stars will be within that range and so the dark sky can be explained.